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Maintenance Optimization

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Maintenance Optimization[1]

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Abstract: Maintenance optimization is the problem of determining cost-optimal maintenance decisions for a system or structure to ensure safe and economic operation. An important concept in maintenance optimization is that of life-cycle costing, where the total costs of design, building, maintenance and demolition are considered over the entire life span of the system or structure in question. In optimizing maintenance, the uncertainties in the time to failure and/or the deteriorating condition should be taken into account. Proper stochastic deterioration models are Markov processes with independent increments (e.g. gamma processes) and Markov decision processes. This paper reviews mathematical decision models to optimize time-based maintenance (e.g. in terms of age and block replacement intervals) and condition-based maintenance (e.g. in terms of inspection intervals). Using renewal theory, optimal maintenance decisions under uncertain deterioration can be determined for which the expected non-discounted cost per unit time or the expected discounted cost over an unbounded time horizon is minimal. Using Bayesian statistics, these optimal maintenance decisions can be adapted on the basis of observations.

Keywords: Age Replacement; Block Replacement; Decision Model; Discounting; Condition-Based Maintenance; Delay-Time Model; Gamma Process; Inspection; Life-Cycle Costing; Maintenance; Markov Decision Process; Opportunity-Based Maintenance; Optimization; Renewal Reward Process; Time-Based Maintenance.

1 Introduction

Because the cost of maintenance is continuously increasing [12], a scientific approach to maintenance optimization is of considerable interest. Maintenance optimization is the problem of determining cost-optimal maintenance decisions for an object (system or structure or one of its component) to ensure safe and economic operation. An important concept in maintenance optimization is that of life-cycle costing (*see* **Life Cycle Costs and Reliability Engineering**), where the total costs of design, building, maintenance and demolition are considered over the entire life span of the object in question. In optimizing maintenance, the uncertainties in the time to failure and/or the deteriorating condition should be taken into account (*see* **Stochastic Deterioration**). Many maintenance models under uncertain deterioration have been developed in various fields of engineering and a large number of papers, mainly focusing on the mathematical aspects, have been published [5, 30, 36, 43, 44, 47, 7, 10, 37, 12, 55, 17]. Herein, a number of probabilistic models for optimizing the life-cycle cost of deteriorating objects are reviewed.

Maintenance optimization techniques are applicable in the design phase and the application phase of the life span of an object. In the design phase, the initial cost of investment has to be balanced against the future cost of maintenance by optimizing the design: the larger the initial resistance, the higher the cost of investment, but the lower the cost of maintenance. In the application phase, the costs of inspection and preventive maintenance have to be balanced against the costs of corrective maintenance and failure by optimizing intervals of inspection and preventive maintenance: the more preventive maintenance is carried out, the higher the costs of inspection and preventive maintenance, but the smaller the costs of corrective maintenance and failure.

2 Maintenance

2.1 Maintenance strategies

Usually, maintenance is defined as a combination of actions carried out to restore a component to, or to "renew" it to, a specified condition in which the component can perform its required functions. Inspections, replacements, perfect repairs, partial repairs (lifetime-extending maintenance) and minimal repairs (restoring the component to its pre-failure state) are possible maintenance actions. Roughly, there are two types of maintenance: corrective maintenance (after failure) and preventive maintenance (mainly before failure). The decision diagram for the choice between corrective and preventive maintenance is given in Figure 1.

Preventive maintenance can be further subdivided into: time-based maintenance carried out at regular intervals of time, use-based maintenance carried out after a fixed cumulative use, operation or load, and condition-based maintenance carried out at times determined by (non-)periodic inspection or continuous monitoring of a component's condition. According to Moubray [31], industrial maintenance changed from corrective to preventive in the fifties; initially time-based maintenance in the sixties (such as equipment overhauls at fixed intervals) and from the seventies on more condition-based maintenance (such as condition monitoring). The above maintenance strategies can be applied to a single component or groups of components.

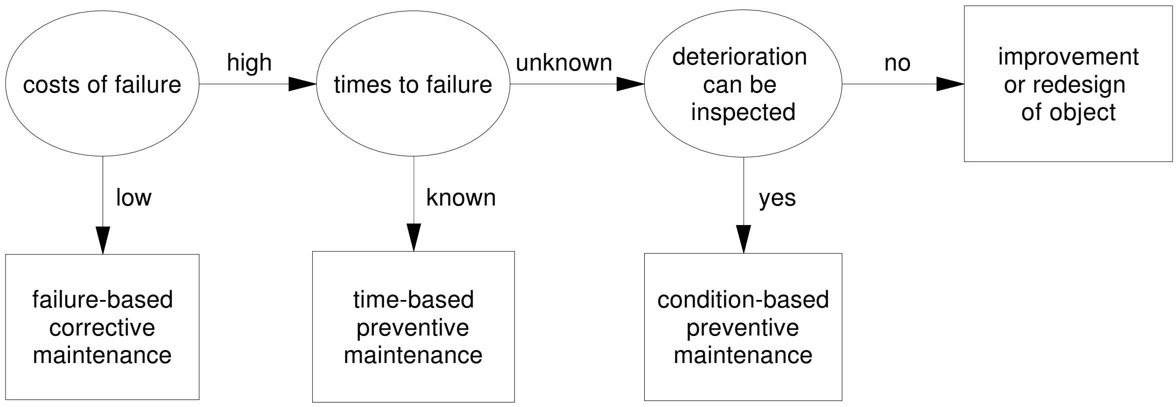

Figure 1: Decision diagram for corrective and preventive maintenance.

In order to determine optimal maintenance decisions under stochastic deterioration (*see* **Stochastic Deterioration**), we distinguish two approaches [39, Chapter 1]: the actuarial approach based on the notion of lifetime (time to failure) and the physical approach based on the notion of failure due to the stress exceeding the resistance (*see* **Stress-Strength Model**). The former is used to model time-based maintenance, the latter to model condition-based maintenance.

2.2 Assumptions, notations, and abbreviations

In the material reviewed below, the following assumptions are made: inspection is perfect in the sense that deterioration will be observed with certainty; inspection, maintenance and replacement take negligible time and don't degrade the component at hand.

In addition, the following notation will be used in the description of the models: c_P is the cost of preventive replacement or maintenance, c_F is the cost of corrective replacement or maintenance and failure, c_I is the cost of a single inspection, c_U is the cost of unavailability per unit time in the event of an unrevealed failure.

Finally, the following standard abbreviations will be used for brevity of presentation: PM (preventive maintenance), CM (corrective maintenance), CBM (condition-based maintenance), EEAC (expected equivalent average cost).

3 Renewal reward processes

Maintenance can often be modelled as a renewal process, whereby the renewals are the maintenance actions that bring a component back into its original condition or "good as new" state (*see* **Renewal Processes**). A renewal process $\{N(t), t \geq 0\}$ is a non-negative integer-valued stochastic process that registers the successive renewals in the time interval $(0, t]$. Let $F(t)$ be the cumulative distribution function of the renewal time $T \geq 0$ and let $c(t)$ be the cost associated with a renewal at time t.

There are three cost-based criteria that can be used to compare maintenance decisions [54, Chapter 11]: (i) the expected average cost per unit time, (ii) the expected discounted cost over an unbounded time horizon, and (iii) the expected equivalent average cost per unit time. Because the planned lifetime of most systems and structures is very long, maintenance decisions may be compared over an unbounded time horizon. For an overview on maintenance optimization over a bounded horizon, see [32].

Using renewal reward theory [40, Chapter 3], the expected average cost per unit time is

$$\lim_{t \to \infty} \frac{E(K(t))}{t} = \frac{\int_0^\infty c(t)\, dF(t)}{\int_0^\infty t\, dF(t)} = \frac{E(\text{cycle cost})}{E(\text{cycle length})}, \tag{1}$$

where $E(K(t))$ represents the expected non-discounted cost in the bounded time interval $(0, t]$. Let a renewal cycle be the time period between two renewals, and recognize the numerator of Equation (1) as the expected cycle cost and the denominator as the expected cycle length (mean lifetime).

In order to account for the time value of money, Samuelson [41] proposed to use the exponential discount function e^{-rt} with constant discount rate $r > 0$ for time $t \geq 0$. Using renewal reward theory with discounting [38, 48], the expected discounted cost over an unbounded horizon can be written as

$$c_0 + \lim_{t\to\infty} E(K(t,r)) = c_0 + \frac{\int_0^\infty e^{-rt} c(t)\, dF(t)}{1 - \int_0^\infty e^{-rt}\, dF(t)} = L(r), \tag{2}$$

where c_0 is the investment cost and $E(K(t,r))$ is the expected discounted cost in the bounded time interval $(0,t]$, $t > 0$. Similar results can be obtained for discrete-time renewal processes [49]. For renewal theory with other types of discounting (such as generalised hyperbolic or gamma discounting), see van der Weide et al. [48].

For the purpose of reserving budget for performing future maintenance actions, it is important to determine how much money these actions cost per unit time while taking the discounting into account. This cost is known as the equivalent average cost per unit time [54, Chapter 11]. The expected equivalent average cost per unit time computed over an unbounded time horizon is defined as

$$\text{EEAC} = \lim_{t\to\infty} \frac{c_0 + E(K(t,r))}{\int_0^t e^{-rx}\, dx} = rL(r). \tag{3}$$

The EEAC per unit time can also be interpreted as a stream of fixed identical cost per unit time sufficient to recover all the necessary discounted cost. As r tends to zero from above, the EEAC approaches the expected average cost per unit time [49]; that is,

$$\lim_{r\downarrow 0} \frac{E(K(t,r))}{\int_0^t e^{-rx}\, dx} = \frac{E(K(t,0))}{t}.$$

The cost-based criteria of discounted cost and EEAC are most suitable for balancing the initial building cost optimally against the future maintenance cost, because the contribution of the initial investment cost is not ignored (as opposed to the expected average cost per unit time). The criterion of the expected average cost per unit time can be used in situations in which no large investments are made (like inspections) and in which the time value of money is of no consequence. Often, it is preferable to spread the cost of maintenance over time and to use discounting. The area of optimizing maintenance through mathematical models based on lifetime distributions was founded in the early sixties [5, 30]. Well-known decision models of this period are the age replacement model and the block replacement model.

3.1 Age replacement

Under an age replacement policy [5, Chapters 3-4], a replacement is carried out at age $k > 0$ (preventive replacement) *or* at failure (corrective replacement), whichever occurs first. Let the time to failure T have a cumulative distribution function $F(t)$. According to Equation (1), the expected average cost of age replacement per unit time is

$$\lim_{t\to\infty} \frac{E(K(t))}{t} = \frac{c_F F(k) + c_P \bar{F}(k)}{\int_0^k t\, dF(t) + k\bar{F}(k)}, \tag{4}$$

where k is the age replacement interval and $0 < c_P \leq c_F$. The optimal age replacement interval k^* is an interval for which the expected average cost per unit time is minimal. Note that preventive maintenance only makes sense for deteriorating components (i.e., having increasing failure rates). According to Equation (2) and Fox [16], the expected discounted cost of age replacement over an unbounded horizon is

$$\lim_{t\to\infty} E(K(t,r)) = \frac{c_F \int_0^k e^{-rt}\, dF(t) + c_P e^{-rk} \bar{F}(k)}{1 - \left[\int_0^k e^{-rt}\, dF(t) + e^{-rk}\bar{F}(k)\right]} = L(r). \tag{5}$$

Note that the replacement model can also be applied for determining an optimal component in the design phase, which balances the initial cost of investment c_P optimally against the future cost of maintenance, by adding c_P to Equation (5). The age replacement model is one of the maintenance optimization models that has been applied most [10, 12]. For example, it has been extended with the possibility of lifetime-extending maintenance in [51].

3.2 Block replacement

Under a block-replacement policy (*see* **Block Replacement**), a replacement is carried out at failure (corrective replacement) *and* periodically at the times $k, 2k, 3k, \ldots$ (preventive replacement). Let the failure times $T_1, T_2, T_3, \ldots,$ be non-negative, independent, identically distributed, random quantities having the cumulative distribution function $F(t)$. The expected average cost of block replacement per unit time when the decision-maker chooses block-replacement interval k is

$$\lim_{t\to\infty} \frac{E(K(t))}{t} = \frac{c_F E(N(k)) + c_P}{k},$$

where $E(N(t))$ is the expected number of failures in $(0,t]$:

$$N(t) = \max\{j \mid S_j \leq t\} = \sum_{i=1}^{\infty} 1_{\{S_i \leq t\}}, \quad t \geq 0,$$

where $S_j = T_1 + \ldots + T_j$, $j = 1, 2, \ldots$, and 1_A denotes the indicator function of the set A. The expected discounted cost of block replacement over an unbounded horizon when the decision-maker chooses block-replacement interval k is

$$\lim_{t\to\infty} E(K(t,r)) = \frac{c_F E(N(k,r)) + c_P e^{-rk}}{1 - e^{-rk}},$$

where $E(N(t,r))$ is the expected number of "discounted failures" in $(0,t]$; that is,

$$E(N(t,r)) = E\left(\sum_{i=1}^{\infty} e^{-rS_i} 1_{\{S_i \le t\}}\right), \quad t \ge 0.$$

The block-replacement policy may be modified by performing a minimal repair when a component fails in the block interval [4]. Assuming the failure rate h before and after the minimal repair to be identical, then $E(N(k)) = \int_0^k h(t)\,dt$ and $E(N(k,r)) = \int_0^k e^{-rt} h(t)\,dt$.

3.3 Inspection to detect failures

The age- and block-replacement models assume failures to be detected immediately (revealed failures). When failures are detected only by inspection (unrevealed failures), then a different maintenance model should be used. For this purpose, we assume that the inspection times are $0 = t_0 < t_1 < t_2 < \cdots$ A renewal is assumed when inspection reveals failure. According to Barlow and Proschan [5, Chapter 4], the expected costs of inspection and unavailability can be written as

$$E(\text{cycle cost}) = \sum_{j=1}^{\infty} \int_{t_{j-1}}^{t_j} \left[c_I j + c_U(t_j - t)\right] \, dF(t) + c_F.$$

and the expected renewal time as

$$E(\text{cycle length}) = \sum_{j=1}^{\infty} \int_{t_{j-1}}^{t_j} t_j \, dF(t).$$

Another inspection model is the delay-time model [8, 3], which assumes the failure process to have two stages: the first stage at which the component is functioning well and the second stage at which the component is still functioning but defective in the sense that failure can be expected soon. Periodic inspection is performed to reveal wether a component is defective or not with inspection interval τ. The time interval between the first moment at which a defect can be noticed until the time of failure is called the delay time and is assumed to have cumulative distribution function G. The occurrence process of defects is assumed to be Poisson with rate λ. Using Equation (1), the expected average cost per unit time can be written as

$$\frac{E(\text{cycle cost})}{E(\text{cycle length})} = \frac{c_I + c_F \int_0^\tau \lambda G(\tau - x)\, dx}{\tau}.$$

3.4 Multi-component maintenance

When production downtime occurs or when other system component failures occur requiring down time, maintenance opportunities may be presented for all components. When such is the case, opportunity-based maintenance models are used. These approaches use either age [11, 9, 42, 24, 23] or block [13, 14] replacement strategies with opportunity times described by a renewal, Markov or Poisson process.

Dependence among components can arise due to economies of scale for maintenance cost, structural relationships, or stochastic dependence of failure times [46]. Accounting for these types of dependencies has also been an important research area in maintenance optimization. Reviews of models for multi-component maintenance can be found in [7, 34] (*see* **Multicomponent Maintenance**).

3.5 Condition-based maintenance

Due to the usual lack of failure data, a reliability approach solely based on lifetime distributions is unsatisfactory. If possible, it is recommended to model deterioration in terms of a time-dependent stochastic process $\{X(t),\ t \geq 0\}$ where $X(t)$ is a random quantity for all $t \geq 0$. The deterioration as a function of time t, $X(t)$, is usually assumed to be a Markov process [5, Chapter 5] (*see* **Markov Processes**). Classes of Markov processes which are useful for modelling stochastic deterioration are discrete-time Markov processes having a finite or countable state space called Markov chains and continuous-time non-decreasing Markov processes with independent increments such as the compound Poisson process and the gamma process. Compound Poisson processes (Gamma processes) are jump processes with a finite (infinite) number of jumps in a bounded time interval (for a sample path of the gamma process, see Figure 2 taken from [50]). Compound Poisson processes are suitable for modelling damage due to sporadic shocks and gamma processes for describing gradual damage by continuous use [50].

3.5.1 Monotone jump process

Modelling the deterioration as a monotone stochastic jump process is especially suited when inspections are involved [50]. In optimizing periodic inspection, the two decision variables are the inspection interval τ and the PM level ρ (Figure 2). Such a policy is called a 'control-limit policy' with the preventive maintenance level called the 'control limit' [55]. The deterioration is inspected periodically (i.e., $t_j = j\tau$, $j = 1, 2, \ldots$) and is regarded as a gamma process. At an inspection, the object can

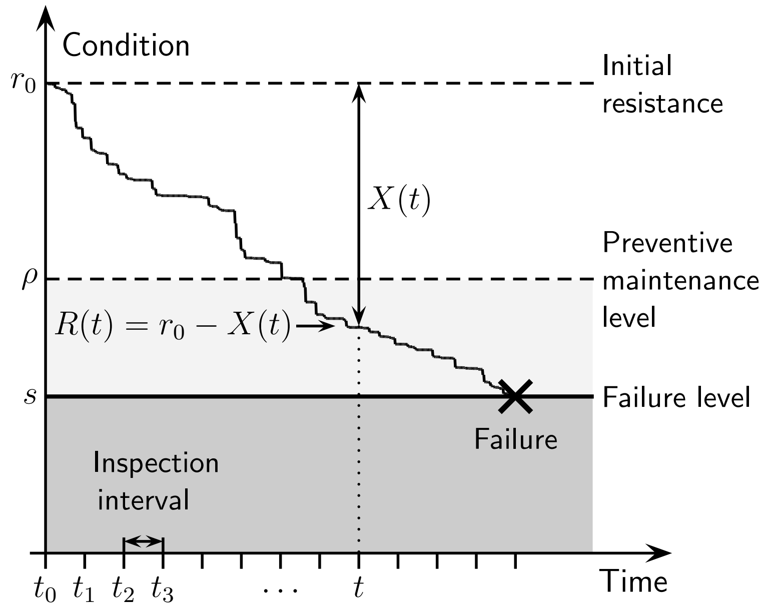

Figure 2: Condition-based maintenance model under gamma-process deterioration.

be in a functional (good), marginal or failed state. If the object is found to be in a functional state, no maintenance is required and only the cost of the inspection is incurred. If it is found to be in a marginal state, the cost of PM is added to the cost of the inspection. For an object in failed state, there are two scenarios: either the failure is immediately noticed at the time of occurrence [27, 35, 53, 33, 25], without the necessity of an inspection, or failure is only detected at the next planned inspection [1, 2, 57]. In the first case, only the cost of CM is incurred and in the second case, the cost of the inspection and unavailability have to be included as well. A renewal can be PM or CM and it brings the object back to its "good as new" condition. The optimal maintenance decision is determined by minimizing the long-term expected average cost per unit time in Equation (1).

When a failure is detected immediately, the expected renewal-cycle cost is the sum of the costs of all inspections during the cycle and either PM or CM:

$$E(\text{cycle cost}) = \sum_{j=1}^{\infty} \left[(jc_I + c_P) \Pr\{\text{PM in } (t_{j-1}, t_j]\} + \right.$$
$$\left. ((j-1)c_I + c_F) \Pr\{\text{CM in } (t_{j-1}, t_j]\} \right]. \qquad (6)$$

In terms of the deterioration process $X(t)$, the event $\{\text{PM in } (t_{j-1}, t_j]\}$ means PM at the jth inspection and is equivalent to $\{r_0 - X(t_{j-1}) \geq \rho, s \leq r_0 - X(t_j) < \rho\}$. The event $\{\text{CM in } (t_{j-1}, t_j]\}$ means CM during the jth inspection interval, which is equivalent to $\{r_0 - X(t_{j-1}) \geq \rho, r_0 - X(t_j) < s\}$. In a similar way, the expected renewal-cycle length is

$$E(\text{cycle length}) = \sum_{j=1}^{\infty} \left[t_j \Pr\{\text{PM in } (t_{j-1}, t_j]\} + E\left(T_F; \text{CM in } (t_{j-1}, t_j]\right) \right],$$

where T_F is the time to failure. Extensions and variations of the CBM model with failures detected immediately include discounting [53, 33], non-periodic inspection [21, 33], and partial or imperfect repair [6].

When a failure is detected only by inspection, the object is renewed when an inspection reveals either that the PM level ρ is crossed while no failure has occurred (PM) or that the failure level s is crossed (CM). The expected cycle cost in Equation (6) should now be extended with the cost of the inspection at which failure is detected and a penalization for the unavailability of the object due to the unrevealed failure being

$$\sum_{j=1}^{\infty} [c_I \Pr\{\text{CM in } (t_{j-1}, t_j]\} + c_U E\left(t_j - T_F; \text{CM in } (t_{j-1}, t_j]\right)],$$

where c_U is the cost of unavailability per unit time. The expected cycle length is

$$E(\text{cycle length}) = \sum_{j=1}^{\infty} t_j \Pr\{\text{PM or CM in } (t_{j-1}, t_j]\}.$$

Extensions and variations of the CBM model with failures detected only by inspection include non-periodic inspection [15, 22], optimal design [45], and damage initiation [52]. A CBM model with deterioration described as a compound Poisson process can be found in Zuckerman [57].

3.5.2 Markov decision process

When a finite or countable state space is assumed, implying that the condition of a component can be in any one of $n \geq 0$ discrete states, a Markov-chain model can be used (*see* **Maintenance and Markov Decision Models**). The condition-based inspection model described in the previous section can also be applied when deterioration is modelled by a finite-state Markov process. See [26] for an application to bridge inspections. However, a more common optimization framework for these processes are the so-called Markov decision processes.

Decisions about an optimal policy for maintenance actions are made on a finite set of actions A and costs $C(i, a)$, which are incurred when the process is in state i and action $a \in A$ is taken. The costs are assumed to be bounded and a policy is defined to be any rule for choosing actions. When the process is in state i at time $t = 0, 1, 2, \ldots$ and an action a is taken, the process moves into state j after one unit of time with probability

$$P_{ij}(a) = \Pr\{X_{t+1} = j | X_t = i, a_t = a\}. \tag{7}$$

Like for regular Markov chains, this transition probability does not depend on the state history. If a stationary policy is selected, then this process is a Markov decision process. A stationary policy arises when the decision for an action only depends on the current state of the process and not on the time at which the action is performed.

Given that the state of the process at times $0, 1, \ldots$ is modelled by a Markov decision process $X_0, X_1, \ldots$ governed by the transition probabilities $P_{ij}(a)$, the optimization of inspection and/or maintenance policies using this process can be performed. For example, when the object is in state i the expected discounted costs over an unbounded horizon are given by the recurrent relation

$$V_\alpha(i) = C(i, a) + \alpha \sum_{j=1}^{n} P_{ij}(a) V_\alpha(j), \tag{8}$$

where $\alpha = 1/(1 + r)$ is the discount factor for one unit of time, r the discount rate and V_α the value function using α. Starting from state i, $V_\alpha(i)$ gives us the cost of performing an action a given by $C(i, a)$ and adds the expected discounted costs of moving into another state one unit of time later with probability $P_{ij}(a)$. The discounted costs over an unbounded horizon associated with a start in state j are given by $V_\alpha(j)$, therefore Equation (8) is a recursive equation. The choice for the action a is determined by the maintenance policy and also includes no repair.

A cost optimal decision can now be found by minimizing (8) with respect to the action a. There are a number of ways to find this optimal solution. One of these is the so-called policy improvement algorithm, where (8) is calculated for increasingly better policies until no more improvement can be made. Also, it is possible to formulate the minimization problem as a linear programming problem. This is used in the Arizona pavement management system [19] and the PONTIS bridge management system [20]. As Golabi [18] illustrates, we can choose to maximize the condition of the road system under a budget constraint or we can minimize the maintenance cost under a minimum safety constraint. This can be achieved by using the original linear programming formulation or with its dual formulation.

The difference between the use of the condition-based inspection model from the previous section and Markov decision processes lies primarily in the fact that the latter approach is used to make a decision about what to do at fixed times $t = 0, 1, 2, \ldots$. The condition-based inspection model has a fixed policy, namely do nothing if the condition is functional, do a preventative repair if the condition is marginal, and do a corrective repair if the object is in a failed condition. Given this policy, the inspection interval and the thresholds for marginal and failed conditions may be optimized. The decision is therefore not what to do, but when to do it.

4 Adaptive maintenance policies

In most cases, the maintenance optimization approaches are developed under the assumption that the parameters of the process are known (or estimated) within a degree of certainty. Adaptive methods for maintenance optimization using a Bayes approach where prior distributions are formulated for distribution parameters and updated as data becomes available have been proposed for only a few basic models. Wilson and Benmerzouga [56] used this approach for analysis of group replacement strategies for parallel components with exponential failure times. Mazzuchi and Soyer [29] propose a Bayes approach for the standard age and block replacement model where the underlying failure time distribution is Weibull. There are several extensions in the literature. In Markov decision processes, Bayes theorem can be applied to update prior distributions on transition probabilities elicited from engineers with inspections [28]. Finally, Kallen and van Noortwijk [25] provide a Bayes analysis for a periodic (imperfect) inspection and maintenance plan where the deterioration process is modeled by a gamma process.

(*See also* **Age Dependent Minimal Repair and Maintenance; Analysis of Recurrent Events from Repairable Systems; General Minimum Repair Models; Group Maintenance; Imperfect Repair; Inspection Policies for Reliability; Multivariate Age and Renewal Replacement; Multivariate Imperfect Repair Models; Nonparametric Methods for Analysis of Repair Data; Optimal Maintenance in Random Environments; Repairable Systems; Replacement Strategies**)

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Related entries: Age Dependent Minimal Repair and Maintenance (EQR129); Analysis of Recurrent Events from Repairable Systems (EQR117); Block Replacement (EQR361); General Minimum Repair Models (EQR116); Group Maintenance (EQR105); Imperfect Repair (EQR107); Inspection Policies for Reliability (EQR101); Life Cycle Costs and Reliability Engineering (EQR095); Maintenance and Markov Decision Models (EQR085); Markov Processes (EQR057); Multicomponent Maintenance (EQR126); Multivariate Age and Renewal Replacement (EQR111); Multivariate Imperfect Repair Models (EQR114); Nonparametric Methods for Analysis of Repair Data (EQR112); Optimal Maintenance in Random Environments (EQR104); Renewal Processes (EQR058); Repairable Systems (EQR348); Replacement Strategies (EQR106); Stochastic Deterioration (EQR062); Stress-Strength Model (EQR350).