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to Military System Data

Cornel Bunea
Department of Engineering Management and Systems Engineering
The George Washington University

Thomas A. Mazzuchi
The George Washington University
Department of Engineering Management and Systems Engineering

Shahram Sarkani
The George Washington University
Department of Engineering Management and Systems Engineering

Hai-Chin Chang
The George Washington University
Department of Engineering Management and Systems Engineering

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Cornel Bunea[1] (*Corresponding author*), Thomas A. Mazzuchi[1], Shahram Sarkani[1] and Hai-Chin Chang[1]

[1] Department of Engineering Management and Systems Engineering

The George Washington University, 1776 G Street, NW, Suite 9, Washington D.C. 20052, USA

cornel@gwu.edu

Tel +1 202 994 7902

Fax +1 202 994 0245

Abstract

This paper focuses on analysis techniques of modern reliability data bases, with an application to military system data. The analysis of military system data base consists of the following steps: clean the data and perform operation on it in order to obtain good estimators; present simple plots of data; analyze the data with statistical and probabilistic methods. Each step is dealt with separately and the main results are presented.

Competing risks theory is advocated as the mathematical support for the analysis. The general framework of competing risks theory is presented together with simple independent and dependent competing risks models available in literature. These models are used to identify the reliability and maintenance indicators required by the operating personnel. Model selection is based on graphical interpretation of plotted data.

Key words: Reliability database, competing risks, failure rate, maintenance indicators

1. Introduction

Huge amounts of operation data have been collected from complex military and industrial systems during the last six decades. Hazardous operating conditions of these systems and their high investment and operating costs require strict guidelines for accurate data collection into reliability databases. These databases have been created not only to gather large amounts of data, but to provide information with regard to main reliability and maintenance indicators, weak components in the system, common cause failures, trends, etc. To meet these demands, support tools for data analysis were also developed.

Competing risks theory was advocated as the main mathematical tool for reliability data analysis [1], [2]. Each event field recorded in the database requires a separate competing risks analysis. For example, several failures modes will compete to end the sojourn life of the component, but only the failure mode that produced the failure will be recorded in the data base for each event. Obviously, such an event also captures information about the others failure modes that did not occur, even if this is not explicitly indicated. In the repair field, competing risks will be associated with the main maintenance actions, i.e. corrective or preventive. The maintenance personnel will always try to avoid a corrective maintenance action, hence the preventive maintenance action will act as a censoring variable for corrective maintenance.

Estimating the underlying failure rate of a competing risk can be quite a challenge without additional assumptions that lead to specific competing risks models. Two main model classes are available in literature: independent and dependent competing risks models. The first class of models assumes that the risks are independent and exponentially distributed [3]. Some derivates of these models take into account other well known distribution functions like weibull, lognormal, etc. The assumption of independence may lead to over-optimistic estimators [4], so dependence between risks should be taken into

account. Early dependent models considered multivariate exponential, weibull or normal distribution functions to model the dependence structure among risks [3], [5]. But all these independent and dependent models did not take into account the characteristics of the analyzed reliability database [6].

Recent independent and dependent competing risks models were developed for the analysis of specific reliability databases. Bedford and Cooke [7] presented several models for the analysis of Swedish nuclear power plants data. In this context, perhaps the simplest dependent competing risks model was introduced: the random signs model. A generalization of this model is captured by the LBL model [8]. Bunea et al. [9] presented a new independent competing risks model for the analysis of Norsk Hydro reliability database – the mixture of exponentials model. Combined, these models proved to cover a larger area of applicability.

The goal of this paper is to apply competing risks models present in literature to the analysis of a military system database collected over 5 years of observation. Main notations and definitions of competing risks will be presented in Section 2. Several independent and dependent competing risks models and their important features are also presented in this section. Section 3 describes the database, and shows how to arrange it in a suitable form for the analysis. The reliability analysis is presented in Section 4. Applicability of various competing risks models is discussed and estimates of distributional parameters are given. Section 5 completes the analysis of the military system database with a maintenance analysis. Main maintenance indicators and plots are given.

Note: The information contained by the military system database will be presented in such a manner that will not violate data confidentiality requirements.

2.   Overview of Competing Risks

Usually, we can reduce the competing risks analysis to two competing risks classes, described by two random variables X and Y, where Y denotes the censoring variable (X can be the minimum of several variables). Hence we observed the least of X and Y, and which of the two variables is observed, i.e. $Z = [\min(X, Y), 1_{\{X<Y\}}]$.

Competing risks data will only allow us to estimate the sub-survival functions $S_X^*(t) = \Pr\{X > t, X < Y\}$ and $S_Y^*(t) = \Pr\{Y > t, Y < X\}$ but not the real survival functions of X and Y. If $S_X^*(t)$ and $S_Y^*(t)$ are continuous at 0 then $S_X^*(0) = \Pr\{X < Y\}$ and $S_Y^*(0) = \Pr\{Y < X\}$. One can see that the subsurvival functions do not have the characteristics of a real survival function; their value at zero is less than one (they add to one at zero), but they are continuously decreasing as time goes to infinity. Additional information may be obtained via the conditional subsurvival functions which are the normalized subsurvival functions:

$$CS_X^*(t) = \Pr\{X > t, X < Y | X < Y\} = S_X^*(t)/S_X^*(0),$$

$$CS_Y^*(t) = \Pr\{Y > t, Y < X | Y < X\} = S_Y^*(t)/S_Y^*(0).$$

The probability of censoring beyond time t, $\phi(t) = \Pr\{Y < X | Y \wedge X > t\} = S_Y^*(t)/(S_X^*(t) + S_Y^*(t))$ seems to have some diagnostic value, enabling us to choose the competing risks model which fits the data. Note that $\phi(0) = \Pr\{Y < X\} = S_Y^*(0)$.

The subdistribution functions for X and Y are defined as:

$$F_X^*(t) = \Pr\{X \le t, X \le Y\} = S_X^*(0) - S_X^*(t),$$

$$F_Y^*(t) = \Pr\{Y \le t, Y \le X\} = S_Y^*(0) - S_Y^*(t).$$

Peterson [10] derived bounds on the survival function of X, $S_X(t)$, by noting that

$$\Pr\{X \le t, X \le Y\} \le \Pr\{X < t\} \le \Pr\{\min(X, Y) \le t\},$$

which results in:

$$1 - F_X^*(t) \ge S_X(t) \ge S_X^*(t) + S_Y^*(t).$$

A restriction to the families of survival function can be obtained by considering bounds to time average failure rate. Recall that the failure rate of X, $r_X(t)$, is given by $r_X(t) = -d(\ln(S_X(t)))$, so the time average failure rate is $\int r_X(t)dt /t = -\ln(S_X(t))/t$. Applying this transformation the Peterson bounds become:

$$-\ln(S_X^*(t) + S_Y^*(0)) /t \leq \text{ is } \int r_X(t)dt /t \leq -\ln(S_X^*(t) + S_Y^*(t))/t.$$

The quantities on the left and the right sides are observable. Similar bounds can be obtained for the survival function of Y, $S_Y(t)$.

2.1 Independent exponential competing risks model

An unique independent model is always consistent with competing risks data (see Tsiatis [11], Weide and Bedford [12]), but exponential survival functions may be usually rejected by the time average failure rate bounds. Cooke [13] related the assumptions of independence and exponentiality to the form of the subsurvival functions. Let X and Y be independent life variables, exponentially distributed with parameters λ_X and λ_Y respectively. Then the subsurvival functions of X and Y are:

• $S_X^*(t) = \lambda_X/(\lambda_X + \lambda_Y) \exp(-(\lambda_X + \lambda_Y) t)$

• $S_Y^*(t) = \lambda_Y/(\lambda_X + \lambda_Y) \exp(-(\lambda_X + \lambda_Y) t)$

One can show that the conditional subsurvival functions of X and Y are equal and exponential distributed with failure rate $\lambda_X + \lambda_Y$ and the probability of censoring beyond time t is constant: $\phi(t) = \lambda_Y/(\lambda_X + \lambda_Y)$.

2.2 Conditional independent model

This model considers the competing risks variables, X and Y, as sharing a common quantity V, and as being independent given V: $X = V + W$, $Y = V + U$, where V, U, W are mutually independent. Hokstadt [14] derived explicit expressions for the case that V, U, W

are exponential distributed. The main result of this model is that the conditional subsurvival functions are equal and the probability of censoring beyond time t is constant.

2.3   *Mixture of exponentials model*

This model was developed for an application to Norsk Hydro data [9]. The main assumptions of this model are that X is drawn from a mixture of exponential distributions, while Y is an exponential distribution and independent of X. This model is a special case of the unique independent competing risks model, but with very important features for data analysis. The main results of this model are: the conditional subsurvival functions are mixture of exponential distribution functions; the conditional subsurvival function of Y lays entirely above the conditional subsurvival function of X; the probability of censoring after time t, $\phi(t)$, is minimum at the origin and increases continuously as a function of t.

2.4   *Random signs model*

Perhaps the simplest dependent competing risks model which leads to identifiable marginal distribution is the random sign censoring model [13]. This model is captured by the following: Let X and Y be life variables with $Y = X - W\delta$, where W is a random variable satisfying $0 < W < X$, and δ is a random variable taking values $\{1, -1\}$, with X and δ being independent.

One can show that $S_X^*(t) = \Pr\{X > t, \delta = -1\} = \Pr\{X > t\}\Pr\{\delta = -1\} = S_X(t)\Pr\{Y > X\} = S_X(t)S_X^*(0)$. Note that $\Pr\{Y > X\}$ and $S_X^*(t)$ can be estimated from data and that under random signs censoring $S_X(t)$ is equal to the conditional subsurvival function of X. In addition, it can be shown that the random signs model is consistent with data if and only if the conditional subsurvival function of X is greater than the conditional subsurvival function of Y for all $t > 0$. In this case the probability of censoring beyond time t is maximum at the origin.

2.5   Model selection

The probability of censoring after time t seems to have an important role in model selection, via graphical interpretation [1]. The following statements follow from the models presented in the previous sections:

1. If the risks are exponential and independent distributed, then the conditional subsurvival functions are equal and exponential distributed, and $\phi(t)$ is constant.

2. If the random signs model holds, then $\phi(0) > \phi(t)$, $t \geq 0$.

3. If the conditional independence model holds with exponential marginals, then the conditional subsurvival functions are equal and $\phi(t)$ is constant.

4. If the mixture of exponentials model holds, then $\phi(t)$ is increasing for all $t \geq 0$.

If $\phi(t)$ changes its monotonicity over the time of observation, and has no maximum or minimum at the origin, then we have no plausible model for coupling X and Y and we will regard them as independent.

3.   Data Description

The military system database contains both failure reports and engineering reports. The engineering report consists of engineering information on component level and engineering information on subcomponent level. At component level, information is collected with respect to component id, component operating modes, maintenance, subcomponents (list of subcomponents). At subcomponent level, information is given with respect to subcomponent id, subcomponent position in the system, manufacturer/design, operating data. From this report, it is possible for the analyst to pool data at subcomponent level and/or at component level in order to obtain better estimates of the reliability and

maintenance indicators [15]. The failure reports give information on system time histories.

content of a failure report is presented below:

Table 1:The content of a failure report

Hierarchical Category	Failure fields	Repair fields
System	Detection date	Code for repair action
Component	Codes for failure detection mode	Date of unavailability
Subcomponent	Codes for failure type	Date start repair
	Failure effect on item	Date of availability
		Number of men used and menhours spend

From the engineering and failure reports, the analyst is able to form a table that

forms the basis for the competing risks analysis. An extract of this table is presented in

Figure 1:

Hierarchical Category			Incident Data										
System/Component			Failure Field					Repair Field					
SYS	Comp	SubComp	FD Code	Detection Date	Failure Effect On Item(FE OI)	FT	AT	Start Un-Av	Start Repair	Start Av	Man	MH	
10504	680718-5G	13EEB	H	2000/1/22	Leaking,right lining break	381	R	2000/1/22	2000/1/22	2000/1/22	2	1.00	
10512	680718-5G	13EEB	E	2000/1/28	Malfunction, break system	242	R	2000/1/28	2000/1/28	2000/1/30	2	2.00	
10675	680718-5G	13EEB	H	2000/2/1	Jammed,right landing gear	135	R	2000/2/1	2000/2/1	2000/2/1	2	1.00	
10477	680718-5G	13EEB	H	2000/2/8	Wore out, left lining break	020	R	2000/2/8	2000/2/8	2000/2/8	2	1.00	
10477	680718-5G	13EEB	H	2000/2/8	Leaking, right lining break	381	R	2000/2/8	2000/2/8	2000/2/8	2	1.00	
10670	680718-5G	13EEB	J	2000/2/8	Leaking, left lining break	381	R	2000/2/8	2000/2/8	2000/2/8	2	0.67	
10488	680718-5G	13EEB	W	2000/2/10	Leaking, right lining break	200	R	2000/2/10	2000/2/10	2000/2/14	2	2.00	
10674	680718-5G	13EEB	R	2000/2/11	Leaking, left lining break	381	R	2000/2/11	2000/2/11	2000/2/11	2	1.33	
10470	680718-5G	13EEB	H	2000/2/11	Leaking, left lining break	381	R	2000/2/11	2000/2/11	2000/2/11	2	0.67	
10671	680718-5G	13EEB	H	2000/2/15	Wore out, left lining break	020	R	2000/2/15	2000/2/15	2000/2/15	2	1.00	

Figure 1: Example of the failure report for the military system data

The "Hierarchical Category" field contains the codes for system, component and

subcomponent. The "Failure" field gives the codes for failure detection, failure type, failure

effect on item, and gives the time of failure. The "Repair" field gives the codes for action

taken, and gives the times when the item was pulled out of service and when it was put

back on service. This field gives also information on the number of men used and the

menhours spent for the maintenance procedure. The failure reports contain also a "Text"

field, which records some observations over failure type, failure cause, repair action etc.

4. Data Analysis

4.1 Data cleaning

The competing risks database proposed for analysis comes from 69 identical military systems, and it was collected over the period 1-22-2000 up to 12-16-2004. This yields 5 years of observation and more than 460 events. The analysis of data can be both statistical and probabilistic [15], and it will be discuss in the next subsection. Nevertheless, the most time consuming operation is "cleaning data". Experience showed that 2/3 of the time of analysis is spent on extracting the most relevant information from the huge amount of information gathered in a reliability database. Out of 460 recorded events in the database, 89 are so called "ties" (multiple entries on the same calendar date). From the competing risks perspective it is important to clarify how many events should be recorded for each field in the database when a tie is present. An example of multiple events recorded on the same date is presented below:

Table 2: Example of multiple events recorded on the same date

System	Component	Sub-Comp	Failure Detection	Detection Date	Failure effect	Failure Type	Action Taken
10477	680718-5G	13EEB	H	2000/2/8	Wore out, left lining break	020	R
10670	680718-5G	13EEB	J	2000/2/8	Leaking, left lining break	381	R
10477	680718-5G	13EEB	H	2000/2/8	Leaking, right lining break	381	R

On date 2-8-2000, due to failure detection mode "H", the subcomponent "13EEB" from component "680718-5G" of system "10477" was found in a failed state due to failure type "020". The maintenance personnel checked the same subcomponent in other systems for similar failures (failure detection mode "J"), and found a failed one in system "10670", but another failure type was present – "381". Afterwards, they went back to initial failed subcomponent in the system "10477" and they found out that failure type "381" was

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present also. The same maintenance action was taken in all events ("R"). Obviously, we have the same subcomponent failed in two different systems caused by different failure modes. One may ask how many events should be considered for each field in the database for this calendar date. Since the systems operate individually, system "10670" induces alone one event for each field in the date base. System "10477" has two entries in the data base, but only the type of failure manifested on the failed subcomponent is different. Hence, this system will induce one event for each field in the database, except for the failure type field with 2 events. Action taken field will record one event for each system. Hence, the total number of events considered for the competing risks analysis will be:

Table 3: Number of events considered for data recorded on 2000/2/8

System	Subcomponent	Failure Detection	Failure Type	Action Taken
2	2	2	3	2

Each "tie" has its own particularities, and must be dealt with separately. The number of events considered for the analysis of each competing risks field will be different for different "ties".

More complications arise when common cause failures are present. Common cause failures are generated by same failure mode manifested on different subcomponents within the very same system (see Table 4).

Table 4:Example of common cause failure

System	Component	Sub-Comp	Failure Detection	Detection Date	Failure effect	Failure Type
10476	680718-5G	13ECB	D	2001/5/28	169	R
10476	680718-5G	13ECA	D	2001/5/28	169	R
10476	680718-5G	13ECC	D	2001/5/28	169	R

There are 12 common cause failure events present in the database: 4 events when two different subcomponents within the same system fail due to the same failure mode, 6 events when three different subcomponents within the same system fail due to the same failure mode, 2 events when four different subcomponents within the same system fail due

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to the same failure mode. Due to the small number of common cause failures and due to the fact that this topic is not the goal of this work, we will neglect the effect of common cause failures from now on. Hence, we will consider one event for each field in the database, except for the subcomponent field where the number of events is equal to the number of failed components.

4.2   Operations on data

Usually, the failure data of a single system is too sparse to apply good competing risks model. Given similarities in design, operating conditions and maintenance regime, the analyst can build a population of similar systems by performing two main operations on failure data: superposition, pooling. To perform these operations on data, a set of assumptions must be made, requiring statistical tests to validate them [1], [15]. One main assumption is homogeneity within the population of systems; this assumption should always be checked with the failure data.

Figure 2 shows the number of failure events for each system in the population. There are in total 373 failure events, distributed unevenly to each system in a range from 1 failure event to 13 failure events. Upper and lower control bounds are also given, and they indicate no outliers among the systems. These bounds are obtained from the well known statistical test called "the log-rank test" (Cox and Lewis, Chapter 9, [16]), considering a 5% significance level for homogeneity of the population of systems. The mean number of failure events per system is also plotted on the graph.

4.3   Survival analysis of the population of systems

Survival analysis of the population of systems investigates the distribution type of the inter-event times, and its main characteristics. The information needed for a survival analysis is: the number of failures per system, and the number of failures per inter-event times for each system and for the superimposed population of systems. Even if the time of
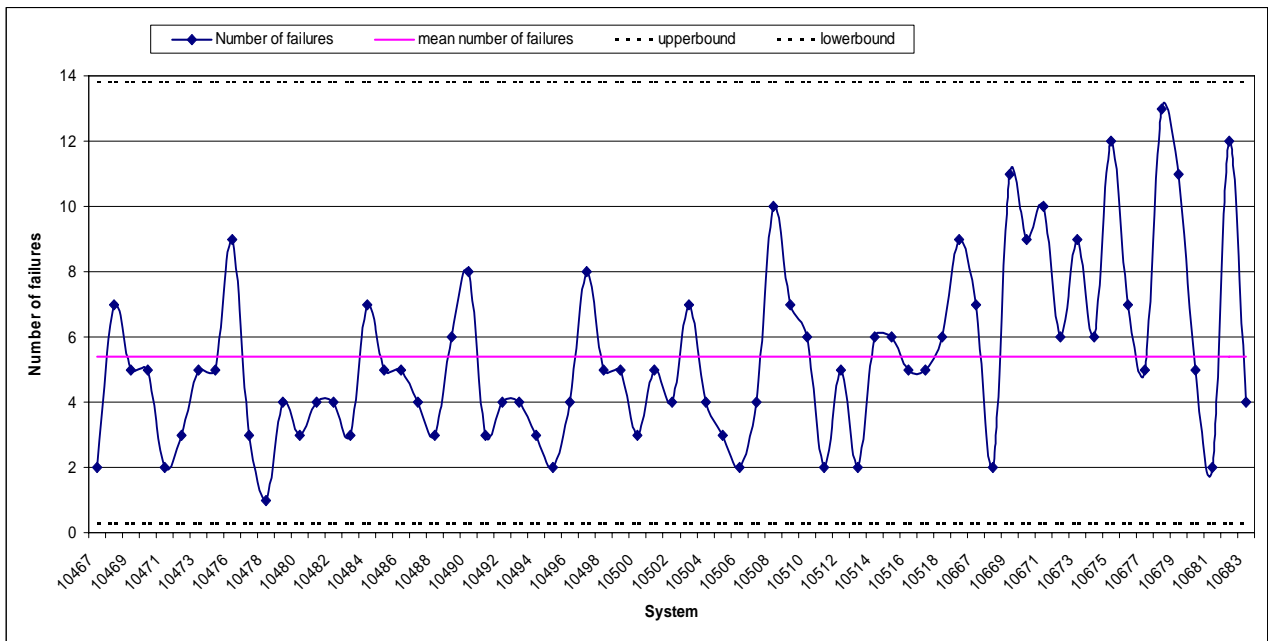
Figure 2: Outliers control chart

observation is 5 years, the number of failures per systems will not be too large (it varies form 1 to 13 failures). If the analyst is interested in a particular subcomponent or failure mode, the number of failures of that subcomponent or failure mode per each system will be even lower. In this case, a two-stage Bayesian model should be used to estimate the mean value of the failure rates together with their 5th, 95th quantiles [15]. In our case, the survival analysis will be performed on the pooled data obtained from the build up population of systems. The amount of data collected will allow us to use classical methods to estimate the desired parameters.

The empirical time average failure rate can be easily plotted from the "cleaned data". A decreasing slope is detected in the shape of the time average failure rate, which is consistent with a mixture of exponential distributions. This can be explained by the human factor involved in the operation of these systems (experienced /inexperienced operators)

and different operating environment, which suggest a mixture of a few systems with very high average failure rates and other systems with low average failure rates.

Empirical and theoretical survival functions are plotted in Figure 4. A mixture of exponentials distribution with mixing parameter p and failure rates λ_1 and λ_2 is considered. One can estimate the parameters using Maximum Likelihood Estimator (MLE) method. Given the observed data $X_1,\ldots, X_n$ the likelihood function is

$$\prod_i^n [p\lambda_1 \exp(-\lambda_1 x_i) + (1-p)\lambda_2 \exp(-\lambda_2 x_i)].$$
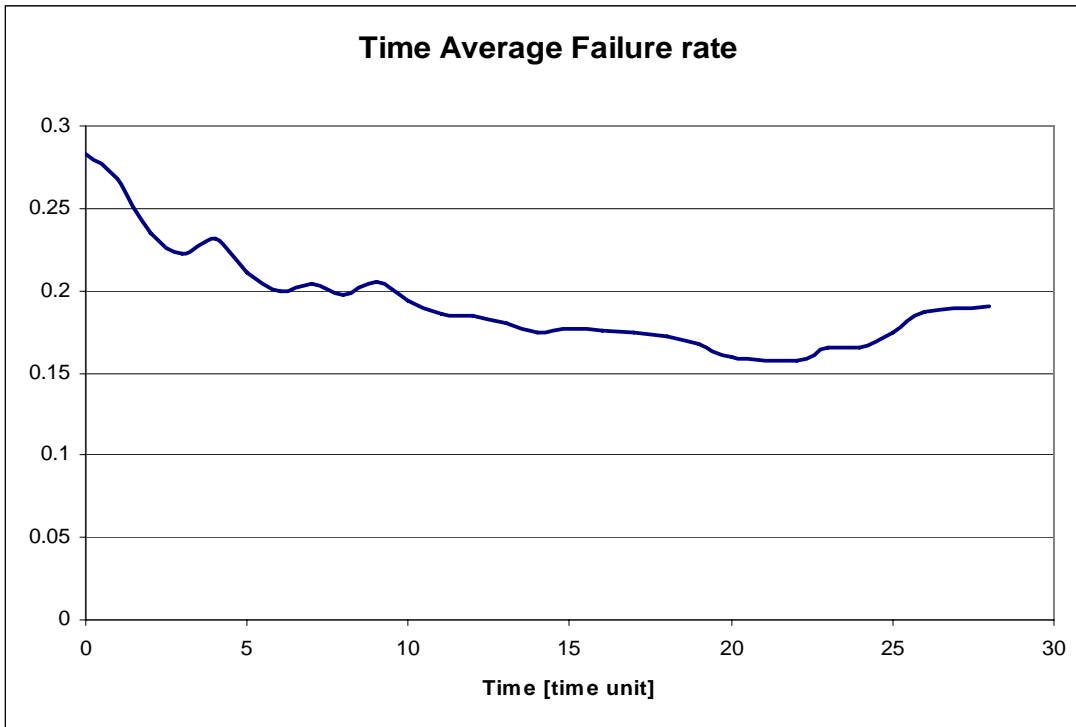
Figure 3: Time average failure rate of the population of systems

Maximizing the logarithm of this function with respect to the parameters yields the following system of equations:

$$\begin{cases} \sum_{i=1}^{n} \dfrac{\lambda_1 \exp(-\lambda_1 x_i) - \lambda_2 \exp(-\lambda_2 x_i)}{p\lambda_1 \exp(-\lambda_1 x_i) + (1-p)\lambda_2 \exp(-\lambda_2 x_i)} = 0 \\[2em] \sum_{i=1}^{n} \dfrac{\exp(-\lambda_1 x_i) - \lambda_1 x_i \exp(-\lambda_1 x_i)}{p\lambda_1 \exp(-\lambda_1 x_i) + (1-p)\lambda_2 \exp(-\lambda_2 x_i)} = 0 \\[2em] \sum_{i=1}^{n} \dfrac{\exp(-\lambda_2 x_i) - \lambda_2 x_i \exp(-\lambda_2 x_i)}{p\lambda_1 \exp(-\lambda_1 x_i) + (1-p)\lambda_2 \exp(-\lambda_2 x_i)} = 0 \end{cases}$$

An analytic solution can not be obtained from the above system of equations. Using the Newton-Raphson method [17], we get the estimated parameters: $p = 0.929$, $\lambda_1 = 0.1604$ [1/time unit] and $\lambda_2 = 0.1342$ [1/time unit]. One can see a good fit of the empirical survival function (Figure 4).
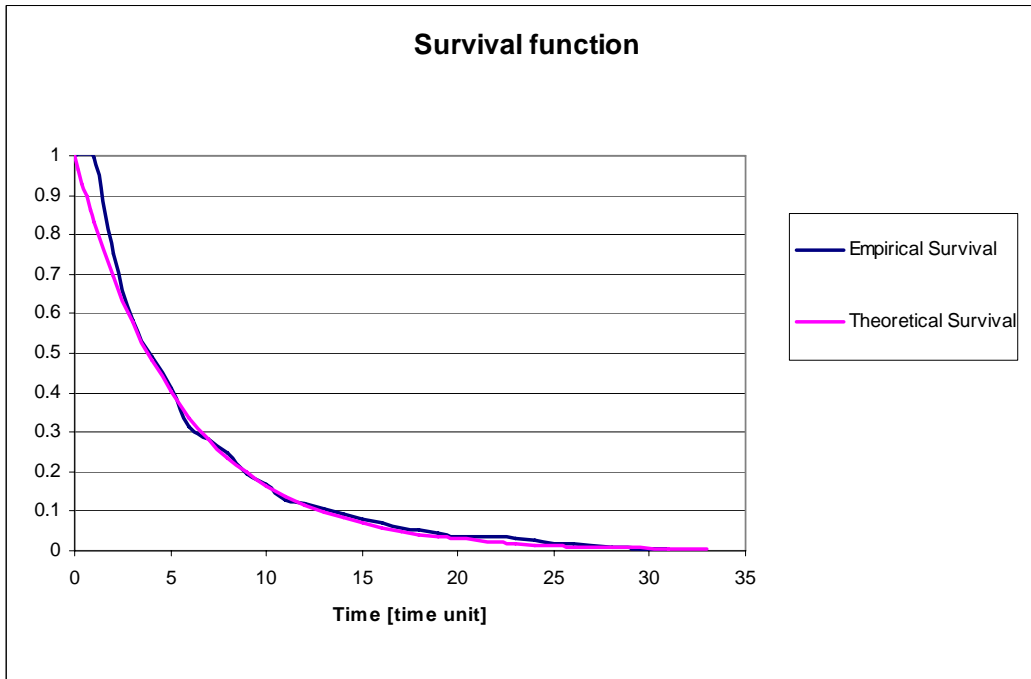

Figure 4: Empirical and theoretical survival functions of the population of systems

4.4   Competing risks analysis for Subcomponent field

Discussion with operating personnel indicated that all "sub-components" can be described as "high value" and "low value". Hence, the maintenance personnel will try to avoid the failure of "high value" subcomponents. Two exhaustive competing risks classes are selected: Risk 1 - corresponding to "high value" subcomponents, and Risk 2 -

corresponding to "low value" subcomponents. The detailed selection is indicated in Table 5 (red/dark color for Risk 1, yellow/light color for Risk 2):

Table 5: Selection of competing risks classes for "subcomponent field"

Sub-component codes	13E99	13EE9	13EA9	13ECB	13E00	13EC0	13EAF	13ECA	13EAA	13EEB	13EAE	13EEC	13EAB	13EAC	13ECC

A Pareto diagram can be used to show the relative frequency of sub-component failures. This diagram shows the design engineer and the maintenance engineer where to focus to for improvement, and shows weak sub-components in the system.
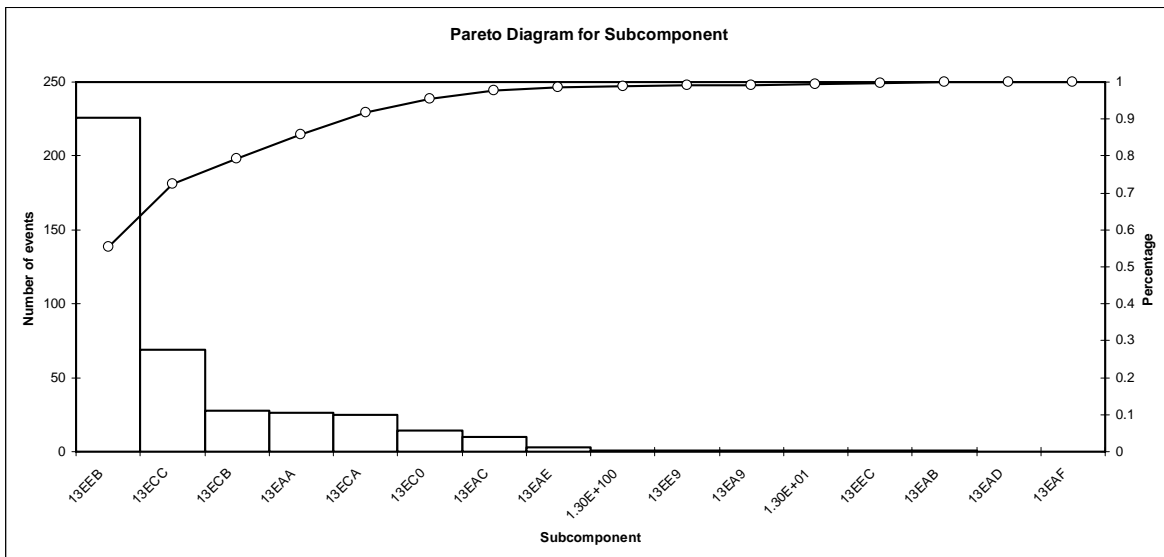

Figure 5: Pareto diagram for subcomponent field

The competing risks graphs show the main competing risks functions presented in section 2: the empirical subsurvival functions, the empirical conditional subsurvival functions, and the empirical probability of censoring beyond time t. The Peterson bounds for the time average failure rate, for both classes of risk, are also plotted. Cooke and Bedford [2] explained how to use such graphical displays in order to choose an appropriate competing risks model to fit data. Bunea et al. [18] proposed the Kolmogorov-Smirnov test

to check whether an independent exponential model is appropriate for the data, against the alternative of the random signs model or the mixture of exponentials model. Once the competing risks model is selected, one can estimate the parameters of the model.

The total number of failures is 407 out of which 13 are "low value" events and 394 are "high value" events. The large number of unwanted events can be explained by a poor maintenance policy. Figure 6 shows that the conditional subsurvival function of Risk 2 dominates the conditional subsurvival function of risk 1, and shows a slight increase in the probability of censoring after time t. Thus, the graphical interpretation of the data might
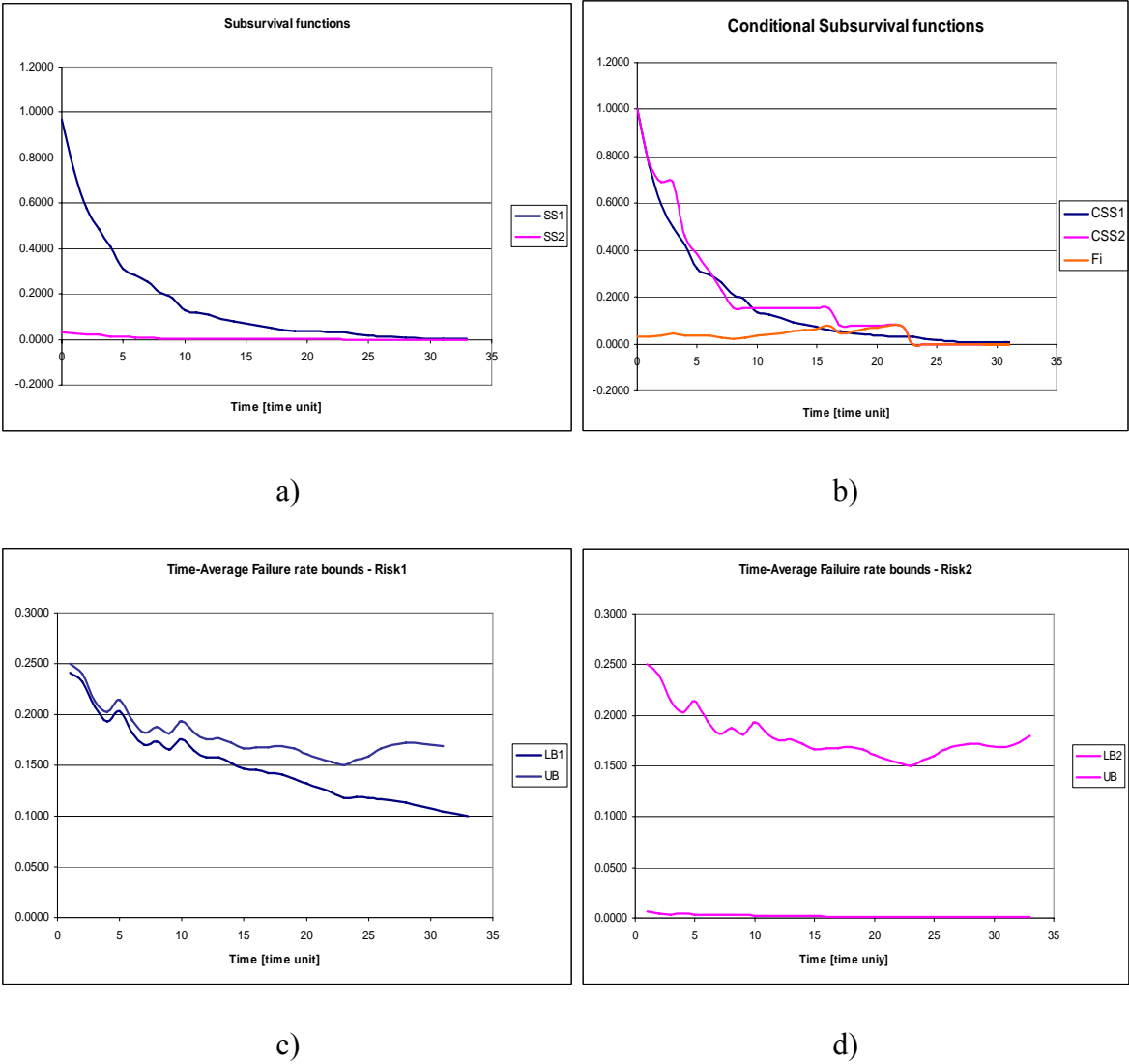

a)

b)

c)

d)

Figure 6: a) subsurvival function; b) conditional subsurvival function and $\phi(t)$; c) and d) time average failure rate bounds for Risk 1 and Risk 2

suggest a mixture of exponentials model to be chosen. Recall that the mixture of exponentials model consists of a mixture of exponential distributions for Risk 1, and an exponential distribution for the censoring risk - Risk 2. This model is also consistent with the Peterson bounds presented in Figure 6: a decreasing time average failure rate and a constant failure rate are allowed for Risk 1 and Risk 2 respectively.

Let λ_{11} and λ_{12} be the failure rates of the mixture of exponential distributions with mixing coefficient p (corresponding to Risk 1), and λ_2 the failure rate of the exponential distribution (corresponding to Risk 2). Using methods presented in [9], the following estimates are obtained: failure rate of Risk 2, $\lambda_2 = 0.005754$ [1/time unit]; failure rates of the mixture, $\lambda_{11} = 2.1306$ [1/time unit], $\lambda_{12} = 0.1682$ [1/time unit], mixing coefficient, p = 0.0355.

4.5   *Competing risks analysis for Failure Detection field*

Two types of engineers are interested in this filed: the risk engineer, and the maintenance engineer. A failure during operating time of the system can have disastrous consequences for the system, and the risk engineer is trying to avoid such events. On the other hand, the maintenance engineer will try to catch a failure during a planned maintenance operation, since the cost of preventive maintenance is lower than the cost of corrective maintenance. However, the selection of the competing risks classes was done from the risk point of view: Risk 1 – "high risk", Risk 2 – "low risk". The detailed selection is indicated in Table 6 (red/dark color for Risk 1, yellow/light color for Risk 2):

Table 6: Selection of competing risks classes for "detection field"

Detection mode codes	A	B	C	D	E	G	H	J	M	Q	R	S	W

A Pareto diagram is used to investigate the main failure detection modes. A number of 55 events are recorded during operating time of the system, which can cause serious damage to the aircrafts. The largest number of failures – 208, is detected after the system completed its job, which can be a plus for the operating personnel since this type of detection is in the "low risk" category.
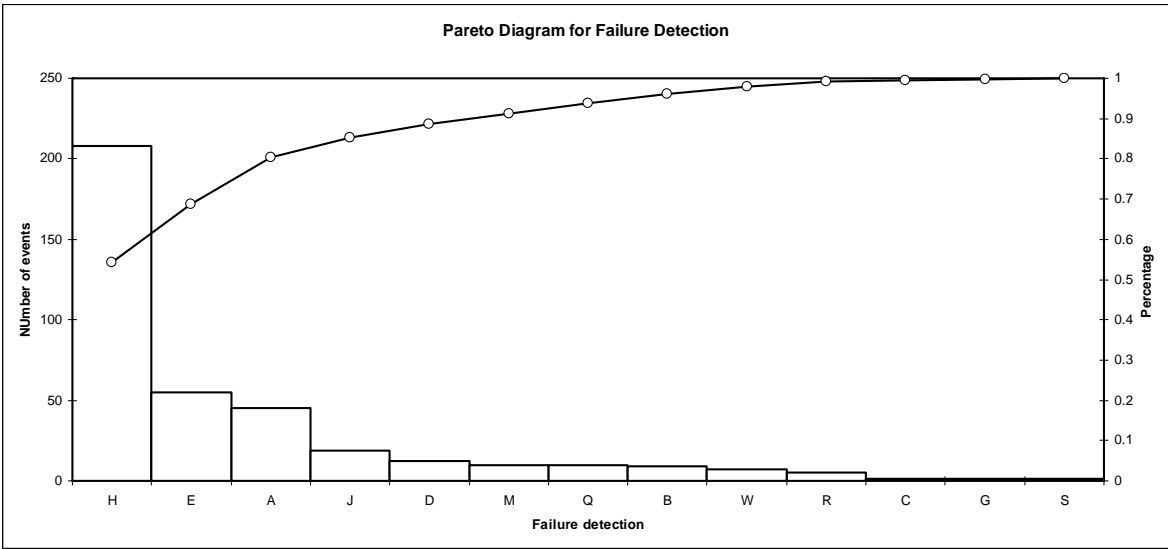

Figure 7: Pareto diagram for failure detection field

The total number of events is 383 out of which 68 are "high risk" and 315 are "low risk". Figure 8 shows that the empirical conditional subsurvival functions are crossing each other once, and that the probability of censoring after time t has an inflexion point. No model is available in the literature for this case, and the risks should be regarded as independent.

The time average failure rate bounds do not allow an exponential distribution or increasing failure rate (IFR) distribution for Risk 2, but allow any type of distribution for Risk 1.
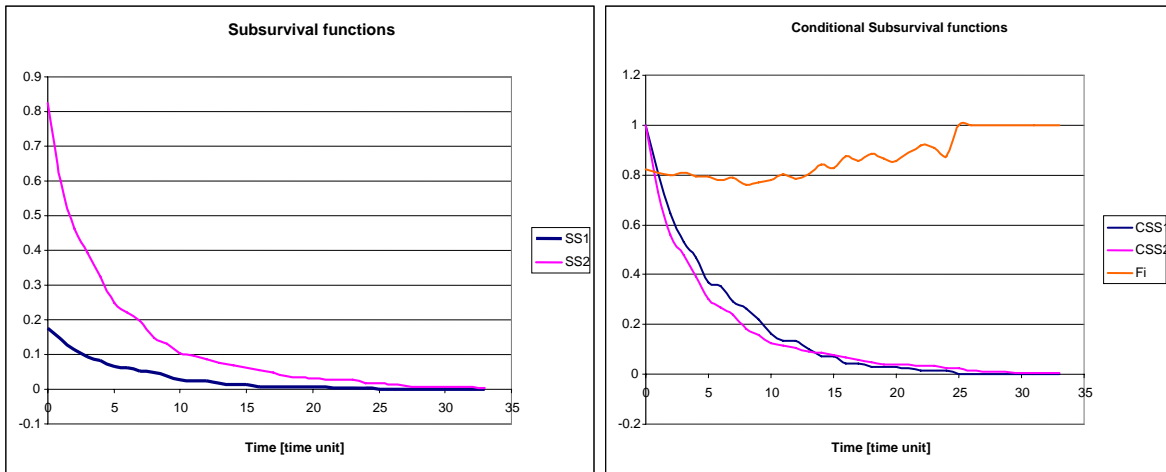
a)   b)
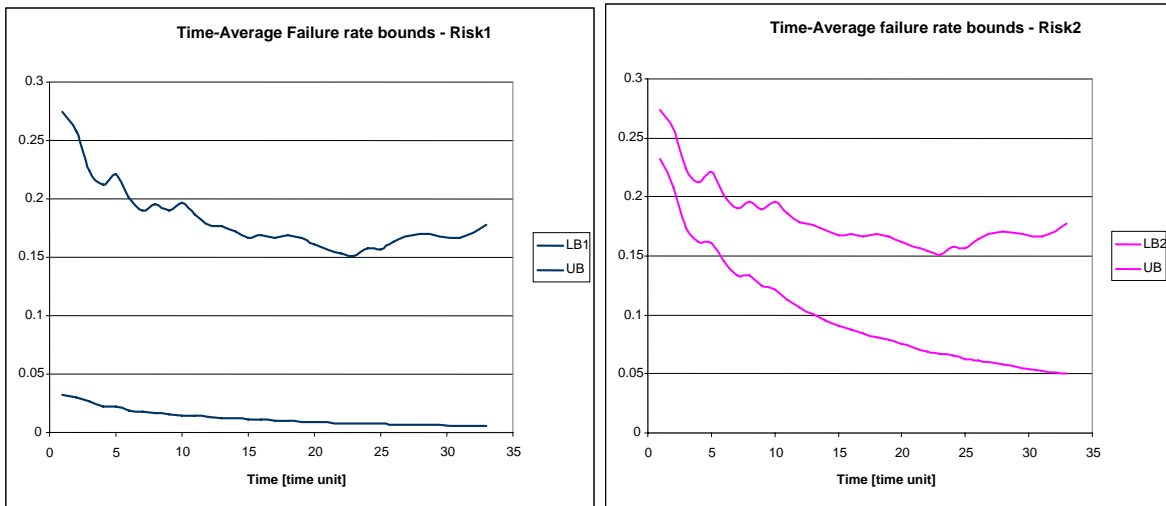
c)   d)

Figure 8: a) subsurvival function; b) conditional subsurvival function and $\phi(t)$; c) and d) time average failure rate bounds for Risk 1 and Risk 2

4.6   *Competing risks analysis for Failure Type field*

The risk engineer is mainly interested in this field. The failure types are classified in two categories: "high risk" and "low risk". The detailed selection is indicated in Table 7 (red/dark color for Risk 1, yellow/light color for Risk 2):

Table 7: Selection of competing risks classes for "failure type field"

Failure type codes	1	8	20	37	64	65	70	80	88	111	127	135	167	169
	180	197	200	223	225	230	242	255	290	334	381	525	553	567
	599	602	607	626	627	710	721	730	786	816	900	901	932	

The Pareto diagram confirms the expectation that the main failure type is "hydraulic oil leaking, internal leaking". This type of failure is often present in complex systems and causes extensive damage [2], [9].

Pareto Diagram for Failure Type

Figure 9: Pareto diagram for failure type field

The total number of events is 399 out of which 283 are "high risk" and 116 are "low risk". The competing risks analysis indicates that the conditional subsurvival function of Risk 1 lays entirely above the conditional subsurvival function of Risk 2, and that the probability of censoring after time t is maximum at the origin. This is consistent with a random sign model.

The time average failure rate bounds do not allow an exponential distribution or increasing failure rate (IFR) distribution for Risk 1, but allow any type of distribution for Risk 2. Under the random signs model, the survival function for Risk 1 is equal to the

conditional subsurvival function of Risk 1. Since the conditional subsurvival functions for both risks can be estimated from data, the survival function for Risk 1 can be obtained. Taking into account the limitations imposed by the time average failure rate bounds of Risk 1, a mixture of exponential distributions with mixing parameter p and failure rates λ_1 and λ_2 is considered. Using Maximum Likelihood Estimator method we get the estimated parameters: p = 0.872, λ_1 = 0.2533 [1/time unit] and λ_2 = 0.1199 [1/time unit].

a)   b)

c)   d)
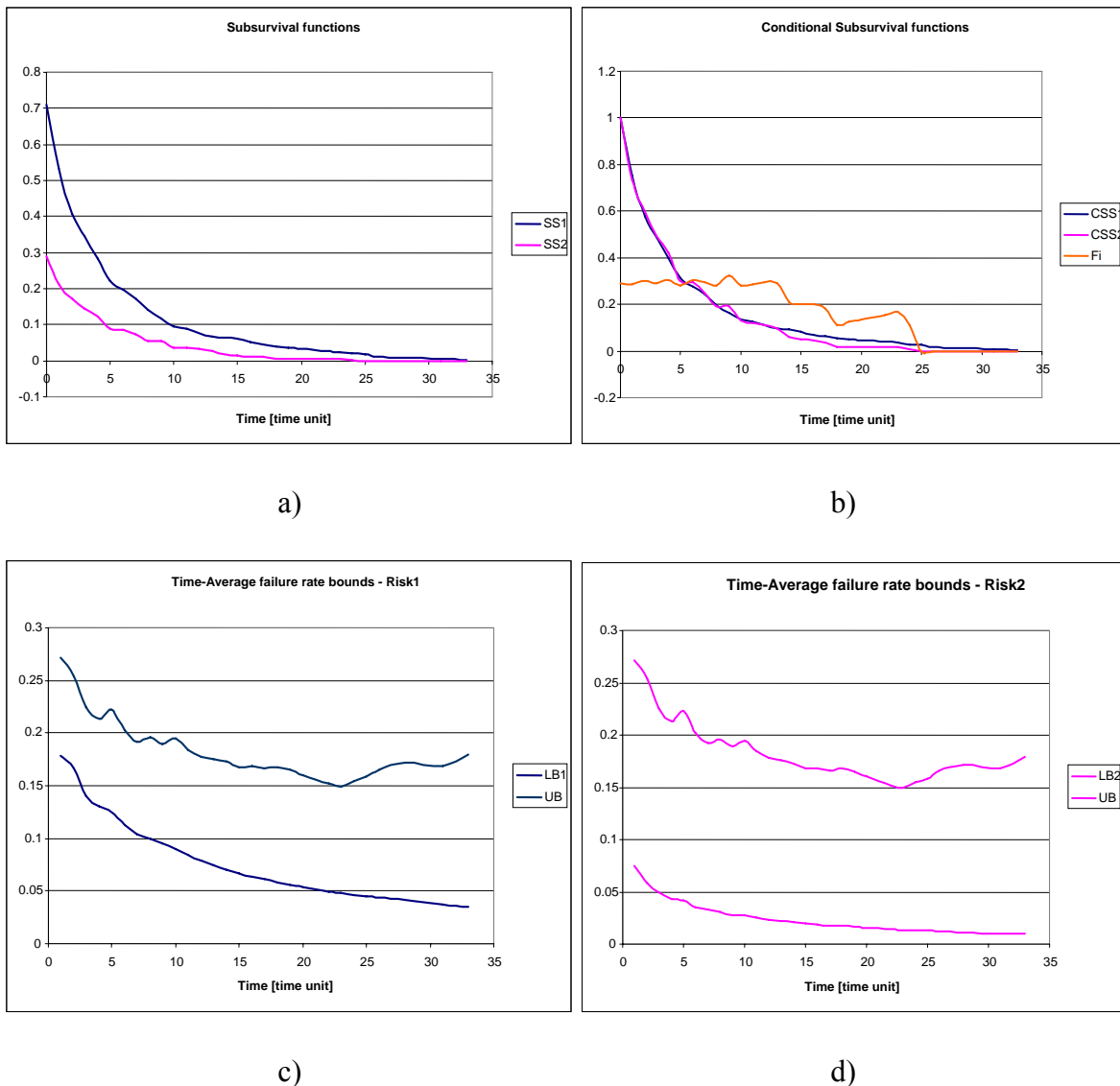
Figure 10: a) subsurvival function; b) conditional subsurvival function and $\phi(t)$; c) and d) time average failure rate bounds for Risk 1 and Risk 2

4.7   Competing risks analysis for Action Taken field

The maintenance personnel indicated no preferences among the action taken activities. However, for the sake of completeness we separate the maintenance actions into two classes: Risk 1 – "R"; Risk 2 – "P", "Q".
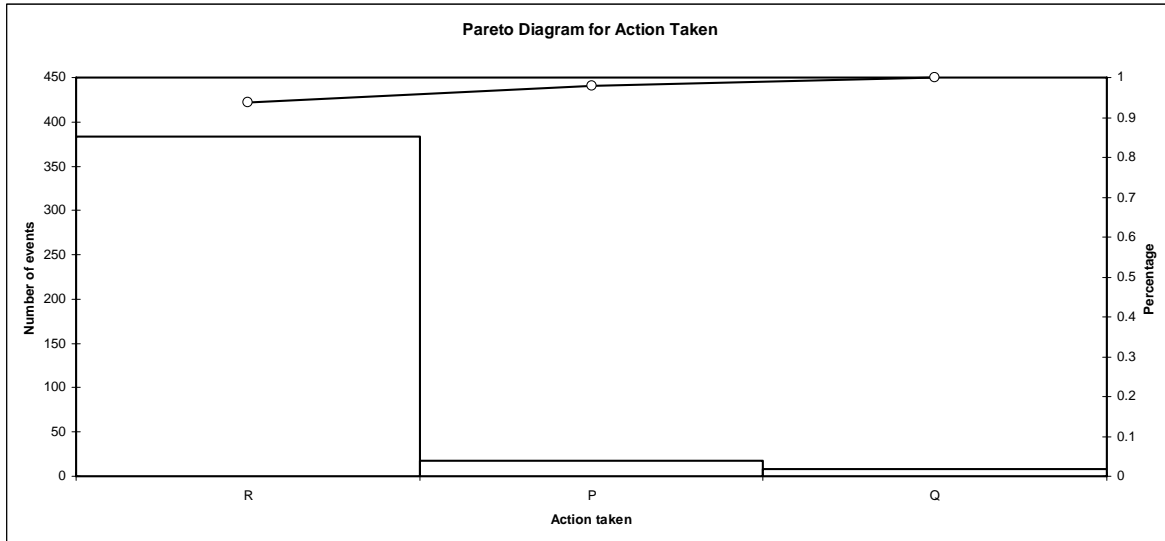

Figure 11: Pareto diagram for action taken field

The total number of events is 408 out of which 383 are "R" type maintenance actions and 25 are other maintenance actions. Figure 12 shows that the empirical conditional subsurvival functions have multiple crossings. No model is available in the literature for this case, and the risks should be regarded as independent. The time average failure rate bounds do not allow an exponential distribution or increasing failure rate (IFR) distribution for Risk 1, but allow any type of distribution for Risk 2.
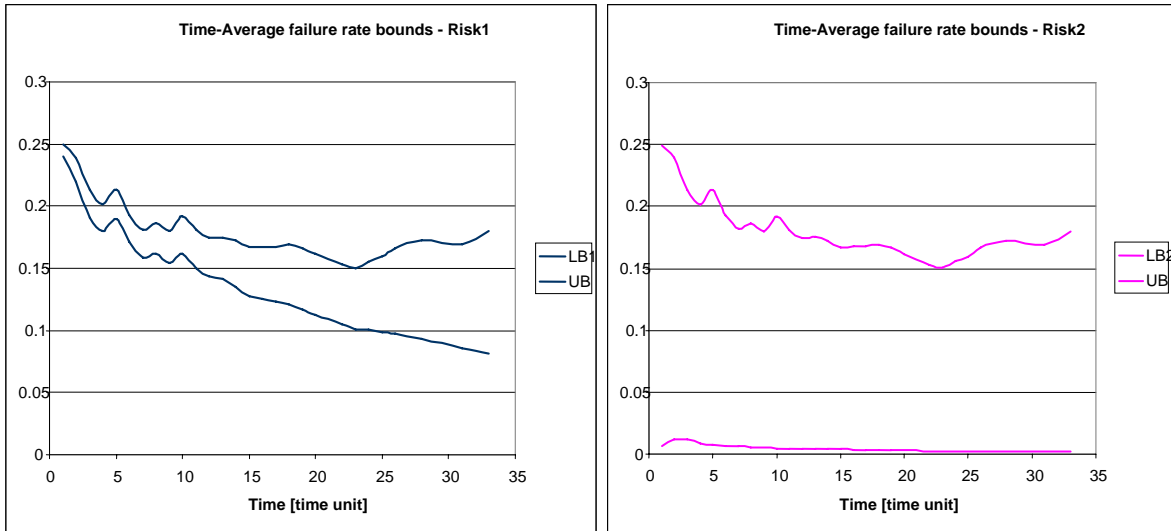
4.8   Maintenance analysis

A maintenance analysis of a repairable system should always be started by plotting an "accumulated number of repair-menhours" graph - N(t) (see [19]). For a constant number of operating systems during the time of observation, the accumulated number of repair-menhours at time t is defined as: N(i) = total repair menhours until the ith repair,
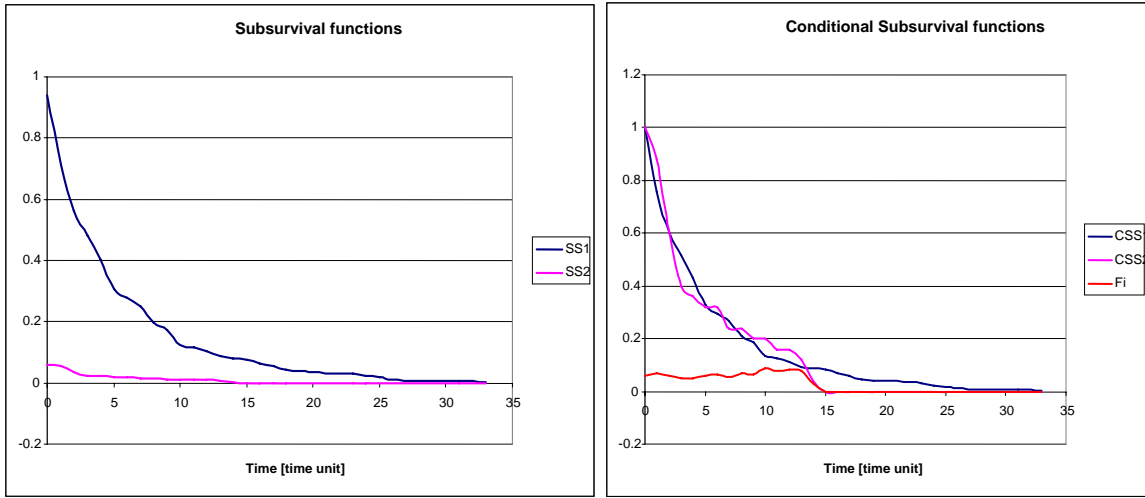
a)   b)

c)   d)

Figure 12: a) subsurvival function; b) conditional subsurvival function and $\phi(t)$; c) and d) time average failure rate bounds for Risk 1 and Risk 2

where the ith repair is the last repair before time t. The slope of the accumulated number of repair-menhours plot between any two points on the graph is an estimator of MTTR between those repair jobs.

Figure 13 illustrates the accumulated number of repair-menhours for the military system data. Note that N(t) plot is convex up to the 200th repair job and concave

afterwards. This indicates an increasing repair rate during the first part of observation period and a decreasing repair rate in the second part. This change in the monotonicity of the repair rate occurs around the 200th repair job, when the largest repair job was performed (179 menhours). The convex part of N(t) corresponds to a good performance of the maintenance team. The concave may be explained by the ageing of the systems combined with the performance of an inadequate maintenance policy.
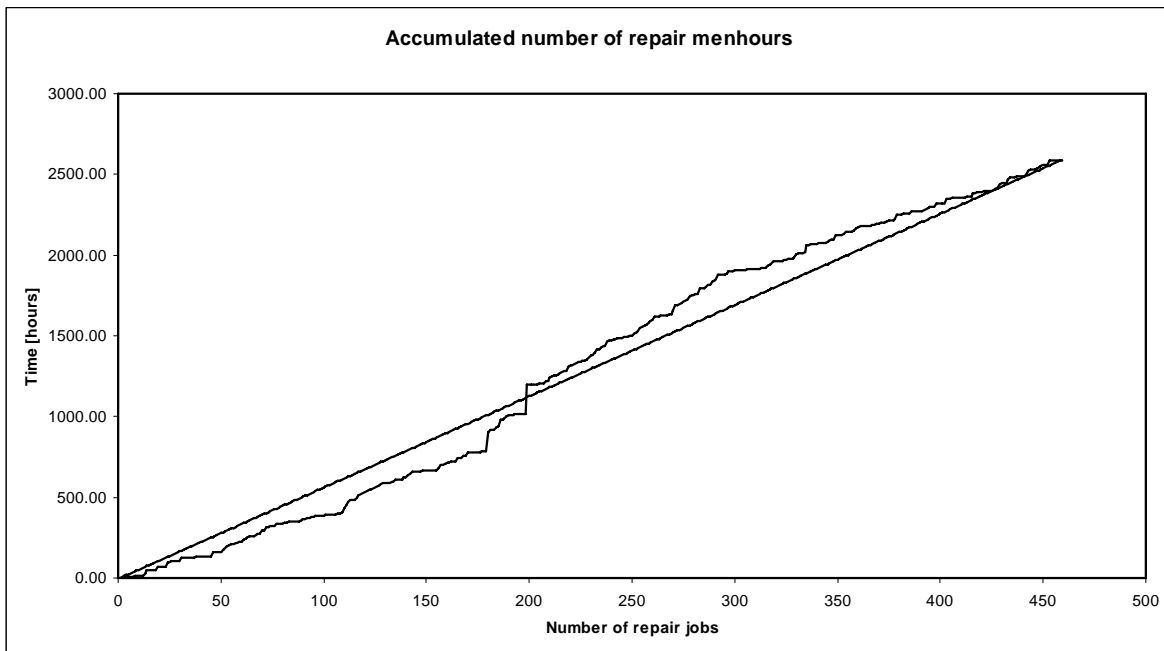

Figure 13: Accumulated number of repair manhours

The yearly fluctuations in the mean number of repair menhours can be directly identified by using a "MMTR during a calendar year" graph. The MTTR during a calendar year is defined as the ratio of the number of repair menhours during a year and the number of repairs during that year.

The MMTR during a calendar year chart does not show big variances from the overall MMTR value (no outliers).

The empirical repair distribution function shows the frequency of repair jobs that took less than t hours. This can give the maintenance engineer an idea of costs of a
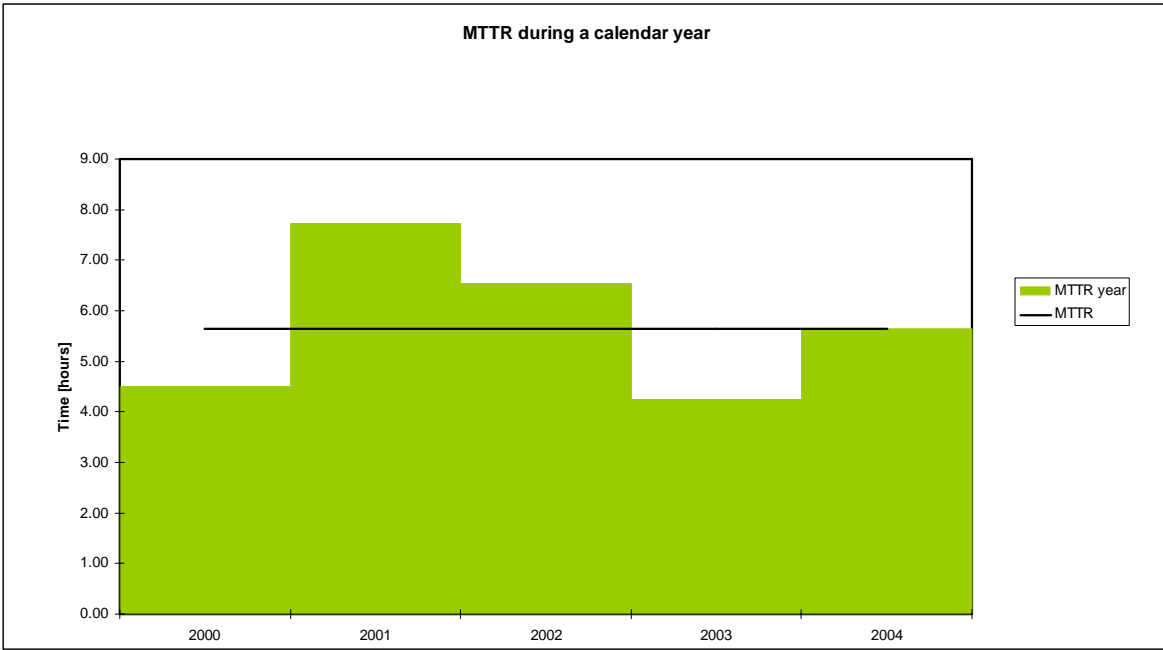
24

Figure 14: Mean time to repair (MTTR) during a calendar year

corrective maintenance job which can support the decision of the total maintenance

planning (e.g. type of maintenance policy to be chosen, optimum replacement times, etc.).
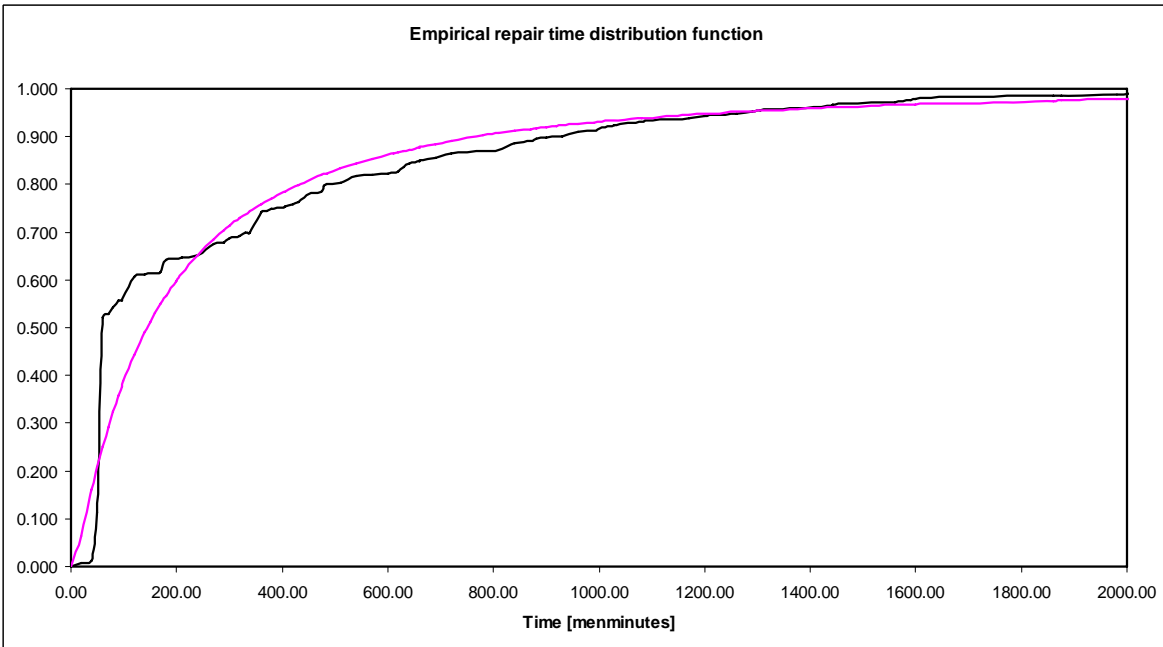

Figure 15: Empirical and theoretical repair time distribution functions

Empirical and theoretical repair time distribution functions are plotted in Figure 15. The most common distribution function used to model the repair times is the lognormal distribution. Using the first and second moments of the lognormal distribution, one can obtain the parameters of the distribution $\mu = 4.9723$ and $\sigma = 1.3053$. One can see a very good fit of the empirical function.

5.   Conclusions

Future research can be performed in several areas. First, more independent and dependent competing risks models should be developed, in order to cover all the possible outcomes of data plotting. One such possible case is when the conditional subsurvival function are crossing each others one time (see the analysis of Failure Detection field), or multiple times (see Action Taken field).

All present models are based on assumptions. One common assumption made in literature is independence between competing risks. This assumption proved to be over optimistic in many cases, as usually data shows a strong dependent relation between competing risks. The research should be extended to models that take into account the dependence structure between risks. One such approach is to consider the dependence structure between risks to be modeled by a copula. Early implementations of these models show the sensitivity of model selection regarding the degree of dependence between risks.

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