

The Pricing of Sovereign Risk Under Costly Information

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We explore the consequences of costly information acquisition on the pricing of sovereign risk. In particular, we consider an environment in which lenders' information acquisition and the sovereign's default decision are jointly endogenous. We find that the model generates state-dependent allocation of investor attention, which has a number of interesting implications: First, it serves as a microfoundation for time-varying volatility in the country-risk spread; second, it implies that model-based estimates of default risk from spread data will be systematically and negatively biased during crisis periods if they fail to account for costly information acquisition; and third, it suggests that sovereign welfare is maximized at an intermediate level of transparency, as opposed to either full or no transparency.

1. Introduction

Yields in sovereign bond markets in emerging economies largely reflect the risk that a domestic borrower may default on foreign creditors since it does not in principle have any reason to care about the well-being of these creditors. But the welfare concern of the sovereign borrower is not the only relevant friction that arises from the international nature of these markets: Information frictions are also likely to play a large role in cross-border financial transactions ([Hatchondo \[2004\]](#) or [Van Nieuwerburgh and Veldkamp \[2009\]](#)). Investors are likely to be far less informed about payoff-relevant events in other countries than they are in their own country. In the information age, information about such events can often be acquired, but

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typically at a cost.

We seek to understand how this costly information acquisition affects the equilibrium pricing of sovereign default risk. In particular, we construct a model in which the sovereign's default and borrowing decision as well as the lenders' acquisition of payoff-relevant information are jointly endogenous. Lenders can costlessly observe some publicly available signals, such output growth and debt levels, but they cannot directly observe other potentially payoff-relevant states, such as populist sentiment, political tides, or specifics regarding sovereign investments and finances. Accurate information regarding these latter states can only be acquired at a cost.

We calibrate the model to Ukraine from 2004-2014 and find that the presence of costly information acquisition generates state-dependence in investor attention on observables: When debt levels are low and output is high, investors are aware that there is little to no gain in terms of accurately forecasting unobservables since default risk is nearly negligible for most of their realizations. Consequently, they save on information costs and acquire less information. However, for high debt levels and low output states, information is more valuable since unobservable variables may substantially influence default risk. Thus, investors are willing to pay more to acquire information about these unobserved states.

Intuitively, investors will start poring over more information sources during crises to carefully study the borrower and its default risk, e.g., professional forecasts, IMF staff reports, credit rating agency reports, and public finance records. This has the flavor of practical techniques employed in the financial sector, where fund managers in charge of multiple portfolios pay relatively limited attention to individual country risk unless publicly available indicators in that country trigger some preset alert.

This state-contingent allocation of attention generates a number of relevant implications for the pricing of sovereign risk. First, it serves as a microfoundation for time-varying or stochastic volatility. During normal times, investors pay little to no attention to payoff-relevant unobservables. Consequently, they assume that these states are at their mean for the purpose of inferring and thus pricing default risk. This implies that bond yields do not respond to realizations of these unobservables during normal times and so volatility is lower. During crises, however, investors pay much more attention to these payoff-relevant unobservables. Therefore they are much better informed about the realization of these unobservables and these observables become priced. This implies that bond yields *do respond* to realizations of unobservables

during crisis times, which increases volatility.

To quantify this channel we develop a model-free metric of time-varying volatility, which we call the *Crisis Volatility Ratio* (CVR). In the data, the CVR is 3.67. We find that the inclusion of costly information acquisition increases the model CVR by 26%, bringing it from 1.68 to 2.12, which is substantially closer to the data. Comparative statics in the model also suggest that falling information costs should imply a substantial and monotone increase in the CVR. We test this implication in the EMBI data over a period of time in which information costs were arguably falling and find it to be true in nearly every country, with the average percent increase in the CVR being 192.7%.

Second, the state-contingent nature of information acquisition has consequences for econometric inference of default frequencies from spread data. The spread on a (short-term) sovereign bond can be decomposed into two components: Default risk and a risk premium for that default risk. When information acquisition is costly, the relative contribution of each to the overall spread will vary endogenously with observable fundamentals. During non-crisis times, investors do not pay attention. However, they are *aware* of their inattention and demand a small risk premium for the volatility associated with unobservables i.e. *taste for risk* varies with observable variables. During crisis times, there is substantial default risk, but little unknown unobservable volatility; investors are better informed and can tolerate the higher risk better. Consequently, the risk premium is relatively smaller, even though absolute risk is higher.

This implies that during non-crisis times, a relatively larger fraction of the spread is comprised of a country-specific risk premium, and that during crisis times a relatively larger fraction of the spread is comprised of pure default risk. This has substantial implications for the inference of default risk from spread data, which is a common practice in the literature ([Bi and Traum \[2012\]](#), [Bocola \[2014\]](#), or [Stangebye \[2015\]](#)). In particular, it implies that a standard sovereign default model without costly information acquisition will tend to *understate* default risk during crises by assuming a risk premium that is too large. We find the median size of this risk premium discrepancy to be about 3.4 percentage points, which implies a median negative bias in the estimate of crisis default probabilities of 29%.

Third, we find a non-monotonicity in sovereign welfare as a function of information costs, which can be interpreted as a measure of transparency. It is better for investors to have to pay some cost to obtain unobservable payoff-relevant information, but that cost ought not be infinite. The intuition is simple and

operates through debt prices. If there is no transparency, then investors demand a greater risk premium during crises, making it more expensive to borrow and service debt precisely when the sovereign is most vulnerable. However, if there is total transparency, the sovereign will be fully exposed to the price volatility that results from normally unobservable shocks. This volatility hurts the risk-averse sovereign. Thus, the optimum lies somewhere in between. Our model actually suggests that the calibrated level of information costs appears to be close to optimal.

It is important to note that we are restricting attention to information frictions that are *country-specific* i.e. those that arise from between a single country and its lenders and does not apply globally. This has substantial implications for our results: For instance, one cannot account for the time-varying volatility by controlling for global metrics such as the CBOE VIX or the P/E ratio, as has been done in the literature (Bocola and Dovis [2015] or Aguiar et al. [2016]); nor can one eliminate the bias in default risk inference using this approach. With regard to the time-varying volatility, our finding corroborates the careful empirical work of Fernández-Villaverde et al. (2011), who find that the bulk of the time-varying interest rate volatility in emerging markets is country-specific rather than global.

A further contribution of this paper is our use of Google’s Search Volume Intensity (SVI) index to identify the cost of information acquisition. The literature has proposed many different measures of information acquisition (Barber and Odean [2008], Gervais et al. [2001], or Seasholes and Wu [2007]), but SVI is one of the few *direct* measures of investor ‘attention.’ Da et al. (2011) demonstrate that it is an effective measure of attention to firm valuation, but to the authors’ knowledge the index has not yet been used to measure attention to a sovereign nation’s financial position.

The actual cost of information acquisition, or alternatively the capacity of the channel agents use to process information (Sims [2003, 2006], Peng and Xiong [2006], or Mackowiak and Wiederholt [2009]), is a tricky but important thing to identify empirically. Most models of ‘rational inattention’ have no direct way of inferring this cost. In this paper, we have a strategy to cleanly identify the cost of information: We match the fraction of quarters in which intense attention is paid to the country. If information is infinitely costly, this fraction will be zero, but if it is free, this fraction will be one. It is clear to see that it will be uniformly increasing in between. We calibrate to match the fraction of periods with information acquisition above the midpoint of its range, which turns out to be around 7.1% of the time.

The remainder of this paper is divided as follows: Section 2 describes the model; Section 3 discusses the data and quantitative implementation of the model as well as the counterfactual analysis; and Section 4 concludes.

2. Model

We consider a model of endogenous sovereign default in the vein of [Eaton and Gersovitz \(1981\)](#). There is a sovereign borrower who issues non-state-contingent debt to a unit mass of foreign lenders. This borrower lacks the ability to commit to repay this debt in subsequent periods and will default if it is optimal to do so ex post.

For simplicity of exposition, we restrict attention to Markov Perfect Equilibria that can be expressed recursively, though the set of equilibria is potentially much larger ([Passadore and Xandri \[2015\]](#)). This is in part for tractability and in part to demonstrate its applicability to the recent, expanding quantitative literature e.g. [Aguar and Gopinath \(2006\)](#), [Arellano \(2008\)](#), [Hatchondo and Martinez \(2009\)](#), or [Mendoza and Yue \(2012\)](#).

2.1. Sovereign Borrower

The sovereign borrower receives a stochastic endowment in each period. Output in period t can be expressed in terms of the sequence of growth rates as follows

$$Y_t = Y_0 \times \prod_{s=1}^t e^{g_s}$$

where Y_0 is given. We assume g_t follows an AR(1) process as follows:

$$g_t = (1 - \rho)\mu_g + \rho g_{t-1} + \sigma_g \epsilon_t$$

where ϵ_t is a standard normal. The sovereign will have time-separable preferences over consumption with CRRA = γ and can issue one period debt abroad. The current stock of debt in period t is denoted B_t .

The only other shock in the model is an additive preference shock to default, M_t , meant to represent any directly unobservable variables governing default behavior e.g. social unrest, political issues, or specifics

of the sovereign's investments and financial situation. It is assumed to be orthogonal to ϵ_s for any s , but this assumption is not strictly necessary.³ In the middle of period t , the shock in period $t + 1$ is realized. The sovereign observes it perfectly and the lenders observe only a noisy signal. We will assume that at the beginning of period t this shock will be given by $\tilde{M}_{t+1} = Y_t^{1-\gamma} \tilde{m}_{t+1}$, where \tilde{m}_{t+1} is distributed normally with mean zero and time-invariant volatility σ_m . The shock is realized between the sovereign's choice of debt issuance in period t and the auction in period t . This timing can be seen in Figure 1.

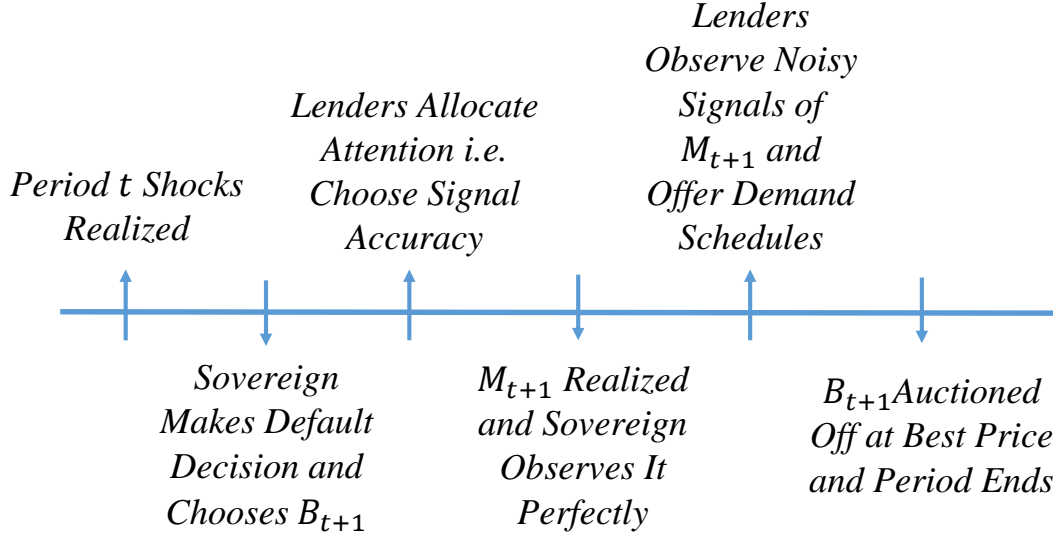


Figure 1: Timing of Events in a Period

As is standard in the literature, we will assume a recursive, Markov-Perfect specification with limited commitment on the part of the sovereign. Given the timing assumption on default, we can express the value of repayment at the beginning of period t as follows:

$$V_t(Y_t, g_t, B_t) = \max_{B_{t+1}} E_{\tilde{M}_{t+1}} \left[\frac{C_t(\tilde{M}_{t+1})^{1-\gamma}}{1-\gamma} + \beta E_{\tilde{Y}_{t+1}, \tilde{g}_{t+1}} \left[\max\{V_{t+1}(\tilde{Y}_{t+1}, \tilde{g}_{t+1}, B_{t+1}), V_{D,t+1}(\tilde{Y}_{t+1}, \tilde{g}_{t+1}) + \tilde{M}_{t+1}\} \right] \right] \quad (1)$$

$$\text{subject to } C_t(\tilde{M}_{t+1}) = Y_t - B_t + q_t(B_{t+1}|Y_t, g_t, \tilde{M}_{t+1})B_{t+1}$$

³If M_t were correlated with any observable variables, then the investors would infer as much information as they could about it from those observables. After this, they would solve a very similar problem to attain any residual information if it proves valuable to attain. Such a model would behave very similarly to our benchmark, so we do not view this assumption as restrictive.

The determination of the issuance price schedule, $q(B_{t+1}|Y_t, g_t, \tilde{M}_{t+1})$, will be discussed momentarily.

We assume that when in default the sovereign is excluded from capital markets for a random number of periods. Further, there are real output costs to default faced by the sovereign in each period of exclusion. These costs are meant to be interpreted as an imposition of international sanctions, a disruption of trade credit, tightening of credit conditions, or any other additional cost faced by the sovereign in default. Under these assumptions, the value of default can be expressed recursively as well.

$$V_{D,t}(Y_t, g_t) = \frac{[Y_t(1 - \psi(g_t))]^{1-\gamma}}{1 - \gamma} + \beta E_{\tilde{Y}_{t+1}, \tilde{g}_{t+1}, \tilde{M}_{t+1}} \left[\phi \max\{V_{t+1}(\tilde{Y}_{t+1}, \tilde{g}_{t+1}, 0), V_{D,t+1}(\tilde{Y}_{t+1}, \tilde{g}_{t+1}) + \tilde{M}_{t+1}\} + (1 - \phi)V_{D,t+1}(\tilde{Y}_{t+1}, \tilde{g}_{t+1}) \right] \quad (2)$$

where $\psi(g_t)$ is the cost of a default in percentage of output. Similarly to [Arellano \(2008\)](#) or [Chatterjee and Eyigungor \(2012\)](#), it is allowed to depend on the current state. In particular, we will assume as those authors do that default is less costly during periods of low growth. ϕ is the Poisson rate at which the sovereign regains access to capital markets. Notice that the preference shock to default only applies in the first period of a default.

2.2. Foreign Lenders

We assume a unit mass of foreign, risk-averse lenders, similar to [Lizarazo \(2013\)](#) or [Aguiar et al. \(2016\)](#). Lenders arrive in overlapping generations and each lender lives for two periods. Each lender is endowed with wealth, w_t , and solves a portfolio allocation problem, deciding how much to invest in risky sovereign debt and how much to invest in a risk-free asset yielding a return, r_t .

Unlike the sovereign, lenders do not receive a perfect signal of the preference shock, \tilde{m}_{t+1} in period t . Rather, they receive a noisy signal of this shock following the sovereign's publicly available choice of debt issuance, B_{t+1} . They can, however, improve their signal by paying for costly information. For tractability, we separate this information acquisition problem from their portfolio allocation decision, calling them Stages I and II respectively. This is similar in spirit to the approach taken by [Gabaix \(2014\)](#) with his two-stage 'sparse max' operator.

All generation- t lenders are ex-ante identical in Stage I. They can pay to acquire costly information regarding \tilde{m}_{t+1} , which we take to be $\rho_{mx,t+1} = \text{corr}(\tilde{m}_{t+1}, \tilde{x}_{t+1})$. The information acquired is given by some increasing and time-invariant function, $I(\rho_{mx,t+1})$. In the benchmark model, we will assume that $I(\cdot)$ is the reduction in entropy in \tilde{m}_{t+1} that comes from knowledge of \tilde{x}_{t+1} i.e. mutual information.⁴ For a bivariate normal distribution, the correlation coefficient is a sufficient statistic to determine the mutual information.⁵

Referring to the timeline in Figure 1, we see that lenders are assumed to know g_t, Y_t and B_{t+1} during this first stage. In this stage, they solve the following problem:

$$\min_{\rho_{mx,t+1} \in [0,1]} E_{\tilde{g}_{t+1}, \tilde{Y}_{t+1}, \tilde{m}_{t+1}, \tilde{x}_{t+1} | g_t} \left[\left(d_t(\tilde{m}_{t+1} Y_t^{1-\gamma}, \tilde{Y}_{t+1}, \tilde{g}_{t+1}, B_{t+1}) - \bar{d}_t \right)^2 \right] + \kappa \mathcal{I}(\rho_{mx,t+1})$$

where $d_t(\cdot)$ is a binary default function that will be defined explicitly in the equilibrium definition, and $\bar{d}_t = E_{\tilde{m}, \tilde{g}_{t+1}, \tilde{Y}_{t+1} | g_t} \left[d_t(\tilde{m} Y_t^{1-\gamma}, \tilde{Y}_{t+1}, \tilde{g}_{t+1}, B_{t+1}) \right]$.

To see the benefit of information acquisition, notice that the variance of the interior expectation is decreasing in $\rho_{mx,t+1}$. Consequently, the variance in the lenders' default forecast can be reduced if the lenders are willing to undergo the disutility of costly information acquisition. When observable fundamentals indicate greater risk of default, this will tend to be optimal; when these observables indicate instead that there is little to no default risk, lenders can save on information costs and accept an imprecise or even independent signal.

Once the correlation has been chosen, each lender receives an idiosyncratic signal, x_{t+1} . This is meant to embody the realistic feature that information acquisition is in fact stochastic. Two lenders could exert the same acquisition effort but reach different primary sources that induce each to value the risky debt slightly differently. This dispersion goes to zero as $\rho_{mx,t+1} \rightarrow 1$. Having received their signal, each lender now solves the Stage II portfolio allocation problem.

In the Stage II problem, a generation- t lender observes her noisy signal x_{t+1} of the sovereign's preference shock and decides how much of her wealth, W_t , to allocate into risky sovereign debt and how much to

⁴This notion of information was developed primarily by Shannon (1958) and applied to economics by Sims (2003, 2006).

⁵None of our key results depend upon this specification of information costs. Any increasing function of $\rho_{mx,t+1}$ will work and we consider alternative information specifications in the appendix as a robustness exercise.

allocate into a risk-free asset yielding a return, r_t . In fact, each lender will offer the sovereign an entire *demand schedule* for debt, $B_{x,t+1}(q_t)$, conditional on their information set. A lender who observes a signal x_{t+1} conditions his optimal behavior on the following state: $s_{l,t} = \{Y_t, g_t, B_{t+1}, \rho_{mx,t+1}, x_{t+1}, \rho_{mx,t+1}\}$.

The lenders' consume only in period $t + 1$ and have CRRA preferences with a risk preference parameter γ_L . Consequently, their demand schedule can be determined by solving the following problem for all $q_t \in [0, \frac{1}{1+r}]$:

$$\max_{B_{x,t+1}} E_{\tilde{g}_{t+1}, \tilde{Y}_{t+1}, \tilde{m}_{t+1} | x_{t+1}, \rho_{mx,t+1}, Y_t, g_t} \left[\frac{\tilde{C}_{x,t+1}^{1-\gamma_L}}{1-\gamma_L} \right]$$

$$\tilde{C}_{x,t+1} = (W_t - B_{x,t+1}q_t)(1+r) + B_{x,t+1} \left[1 - d_{t+1}(\tilde{m}_{t+1}Y_t^{1-\gamma}, \tilde{Y}_{t+1}, \tilde{g}_{t+1}, B_{t+1}) \right]$$

The solution to this problem is a demand schedule $B_{x,t+1}^*(q, B_{t+1}, x, g_t, Y_t)$, which lender x offers to the sovereign.

After the lenders offer the sovereign their heterogeneous demand schedules, the sovereign chooses an issuance price.⁶ Naturally, he will choose the largest price subject to the restriction that he must in fact issue B_{t+1} in the aggregate. We can use this restriction to construct the pricing schedule as follows: For any state $\langle m_{t+1}, g_t, Y_t \rangle$ and for any potential issuance, B_{t+1} , let

$$q_t(B_{t+1} | m_{t+1}, g_t, Y_t) = \sup \left\{ q \mid \int_x B_{x,t+1}^*(q, B_{t+1}, x, g_t, Y_t) f_{x|m_{t+1}}(x) dx = B_{t+1} \right\} \quad (3)$$

Generally this set will be degenerate. Under the assumptions so far, the distribution $f_{x|m_{t+1}}(x)$ will be normal with a known mean and variance. Depending on the choice of $\rho_{mx,t+1}$, the mean can depend on the realization of m_{t+1} . Thus, the market price can move with m_{t+1} even though it is unknown to the lenders at the time they offer their demand schedules.

Having described the model, we can now define our equilibrium:

Definition 1. *A Markov Perfect Equilibrium is a set of functions, $\{V_t(Y_t, g_t, B_t), A_t(Y_t, g_t, B_t), V_{D,t}(Y_t, g_t), q(B_{t+1} | Y_t, g_t, \tilde{M}_{t+1})\}_{t=0}^\infty$ such that*

⁶Notice that upon observing the issuance price, lenders will be able to infer M_{t+1} . At this point, however, it is too late for them to make use of this information since all decisions will have been taken already.

1. $V_t(Y_t, g_t, B_t)$ and $V_{D,t}(Y_t, g_t)$ solve Recursions 1 and 2 and imply the policy $B_{t+1} = A_t(Y_t, g_t, B_t)$.
2. $q(B_{t+1}|Y_t, g_t, \tilde{M}_{t+1})$ defines aggregate bond demand given by Equation 3 when $d_t(M_t, Y_t, g_t, B_t) = \mathbf{1}\{V_t(Y_t, g_t, B_t) < V_{D,t}(Y_t, g_t) + M_t\}$.

2.3. Solving the Model

We will stationarize this model by dividing the sovereign resource constraint in every period by Y_t , and the value function by $Y_t^{1-\gamma}$. This delivers a convenient recursive structure which is independent of both time and the level of output. We will denote $b' = B_{t+1}/Y_t$ and $c = C_t/Y_t$.

$$v(g, b) = \max_{b' \geq 0} E_{\tilde{m}} \left[\frac{c(\tilde{m})^{1-\gamma}}{1-\gamma} + \beta E_{\tilde{g}} \left[\max\{e^{\tilde{g}(1-\gamma)} v(\tilde{g}, b'), e^{\tilde{g}(1-\gamma)} v_{D,t+1}(\tilde{g}) + \tilde{m}\} \right] \right]$$

$$c(\tilde{m}) = 1 - b e^{-g} + q(b'|g, \tilde{m})b'$$

The value of default is scaled similarly, yielding

$$v_d(g) = \frac{(1 - \psi(g))^{1-\gamma}}{1-\gamma} + \beta E_{\tilde{g}, \tilde{m}} \left[\phi \max\{e^{\tilde{g}(1-\gamma)} v(\tilde{g}, b'), e^{\tilde{g}(1-\gamma)} v_{D,t+1}(\tilde{g}) + \tilde{m}\} + (1 - \phi) e^{\tilde{g}(1-\gamma)} v_d(\tilde{g}) \right]$$

This stationarization implies that we can express the default policy function using only stationarized model objects, since Y_t does not influence the default decision once g_t is known.

$$d(m, g, b) = \mathbf{1}\{e^{-g(1-\gamma)} v(g, b) < e^{-g(1-\gamma)} v_D(g) + m\}$$

With this simplification, we can stationarize the lender's problem version and decompose the joint expectation into a conditional and unconditional one:

$$\min_{\rho_{mx} \in [0,1]} E_{\tilde{x}, \tilde{g}} \left[E_{\tilde{m}|\tilde{x}, \tilde{g}} \left[(d(\tilde{m}, \tilde{g}, b') - \bar{d})^2 \right] \right] + \kappa \mathcal{I}(\rho_{mx})$$

The second stage of the lenders' problem also stationarizes as well under the assumption that $W_t = wY_t$.

$$\max_{b'_x} E_{\tilde{m}|x, \tilde{g}} \left[\frac{\tilde{c}_x^{1-\gamma_L}}{1-\gamma_L} \right]$$

$$\tilde{c}_x = (w - b'_x q)(1+r) + b'_x [1 - d(\tilde{m}, \tilde{g}, b')]$$

3. Quantitative Analysis

To determine the impact that costly information acquisition has on the pricing of sovereign risk, we calibrate the model to match a set of empirical moments from Ukraine from 2004-2014. We choose this time period since it is the only period for which Google search data, which is key in our approach, is available. We choose Ukraine since its macroeconomic properties during this time are rather similar to Argentina during the 1990's, which is the canonical calibration choice for models in this vein ([Aguiar and Gopinath \[2006\]](#) or [Arellano \[2008\]](#)), and thus the model will have little difficulty matching the key moments. In contrast, many Latin American economies during the 2000's exhibit substantially lower growth and spread volatility than in the decades prior. Further, Ukraine was at the heart of several news cycles over the course of this cycle, including a political upheaval during the Russo-Georgian War in 2008 and the Russian acquisition of Crimea in 2014.

We will find that the model endogenously generates acquisition of information that is contingent on observable states. The intuition is the following: During non-crisis times, information is not particularly valuable to a foreign investor. Thus, investors do not acquire it and price sovereign debt assuming unobservables to be at their average. During crisis times, however, information is highly valuable. Consequently, it is both acquired and priced, which increases the volatility of country spreads.

This generates a number of interesting features including time-varying volatility, state-contingent risk premia that are relevant for econometric inference, and some novel welfare results on transparency for the sovereign borrower.

A novelty in our approach is our ability to directly identify information acquisition using Google SVI, rather than using a model-based inference based on some moment of the spread distribution ([Da et al. \[2011\]](#)). Being able to directly observe and parameterize information acquisition allows for sharp counterfactual analysis. We will discuss the process of identification in the next section.

We solve the model using value function iteration on a discrete grid.

3.1. Data and Calibration

We take data from three primary sources: First is the JP Morgan Emerging Market Bond Index (EMBI) database; second is the World Bank; and third is Google Trends' Search Volatility Index (SVI).

To obtain the output process, we estimate via MLE an AR(1) process $(\rho_g, \mu_g, \sigma_g)$ on real hryvnia-valued GDP growth for Ukraine from 2004-2014 at a quarterly frequency. We restrict attention to this time period for consistency since Google's SVI is only available starting in 2004.

We choose the form of output costs in autarky similarly to [Chatterjee and Eyigungor \(2012\)](#), with a constant term and a curvature term.

$$\phi(g) = \phi_0 + \phi_1 \times g$$

where ϕ_0 and ϕ_1 are both positive. This implies that when growth is negative, the proportional default costs are below ϕ_0 and when growth is positive these costs are above ϕ_0 .

We further assume that the risk-free rate is fixed at 1% quarterly; that both the sovereign and the lenders exhibit constant relative risk-aversion preferences with CRRA=2, which is standard; and that $\theta = 0.083$, which is an estimate from [Mendoza and Yue \(2012\)](#).

To match the information costs, we match the variability of information acquisition in the model and in the data. Following [Da et al. \(2011\)](#), we measure information acquisition in the data by employing Google search trends for terms for which investors are likely to search. In particular, we consider Google's *Search Volume Index* (SVI), which is a scalar measure between 0 and 100 over a given time period that measures the intensity of Google searches for a given term. 100 is always normalized as the maximum value and consequently is always contained in any requested series.

The process of identification is as follows: First, we transform the raw SVI series into the *Abnormal Search Volume Index* series (ASVI), as suggested by ([Da et al. \[2011\]](#)). The ASVI is meant to capture a notion of paying 'extra' attention to a certain event or item in a period t . In any period t it is computed as follows:

$$ASVI_t = \log(SVI_t) - \log(\text{Median}(\{SVI_{t-9}, SVI_{t-8}, \dots, SVI_{t-1}\}))$$

In the benchmark, we compute the ASVI for the search term ‘Ukraine IMF,’ since investors are more likely to be interested in IMF staff reports than average search users. Before we perform the ASVI transform, we average monthly data from 2004-2016 into quarterly data.⁷ We consider other terms for robustness in the appendix. The fully transformed series can be found in Figure 2.

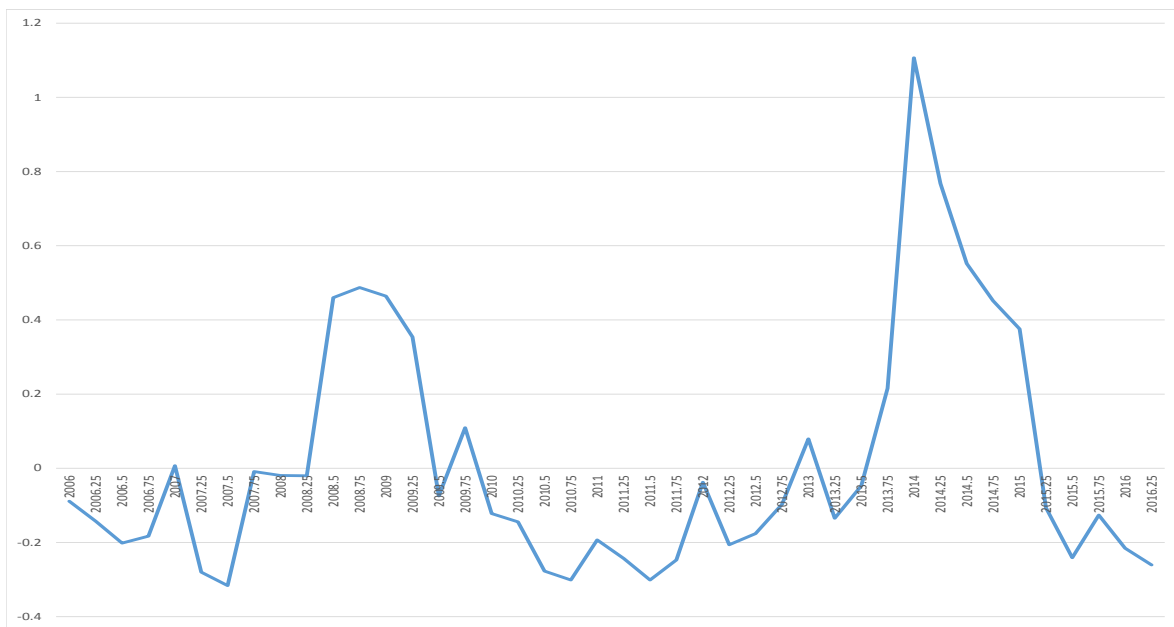


Figure 2: Quarterly ASVI for Search Term ‘Ukraine IMF’

After computing the ASVI, we define an information threshold $\zeta = 0.5 \times \max_t\{ASVI_t\}$. Our target then becomes the fraction of quarters spent above this threshold. Assuming that the SVI is stationary, the unconditional mean of the ASVI should be zero. Thus, the crossing of the threshold is capturing the frequency of large, positive, and relatively discrete jumps in attention. This is the sort of behavior that the model predicts and so it is easy and natural to match.

For our data, this ratio turns out to be about 7.1%. It captures the two visible ‘peaks’ around mid-2008 and early 2014. The former was largely tied to a political crisis in 2008 often thought to have been sparked by Russia’s armed conflict with nearby Georgia; the latter to its own conflict with Russia and annexation

⁷The results are robust to applying the ASVI transform before averaging.

of the Crimean peninsula.

We calibrate six parameters, $\{\beta, \phi_0, \phi_1, w, \sigma_m, \kappa\}$, to roughly match six moments in the data: Average annual spread, annual spread volatility, annual spread skewness, debt-service to GDP, annual default frequency, and fraction of time in which information acquisition (IA) is above 50% of its max value. These parameters are given in Table 1; the fit is not perfect, but it's quite close. Each parameter is roughly speaking identified by its corresponding target moment, though there are cross-partial effects.

Table 1: Calibration

Parameter	Value	Target Description	Data Moment	Model Moment
β	0.90	Annual Default Frequency	1.0%	1.1%
ϕ_0	0.0329	Average Debt-Service-to-GDP Ratio	12.6%	12.0%
ϕ_1	0.85	Spread Volatility	5.5%	3.9%
w	1.0	Average Spread	6.4%	5.9%
σ_m	0.014	Spread Skewness	2.32	2.42
κ	0.000264	Fraction of Quarters with $IA > 50\% \times \max(IA)$	7.1%	9.8%

3.2. Model Behavior

The model itself behaves quite similarly to a standard sovereign default model along many dimensions. For instance, Figure 3 gives the demand functions in equilibrium. Equilibrium demand for debt exhibits a simple, downward sloping feature that looks remarkably similar to [Arellano \(2008\)](#) despite the added complexity of signal extraction and lender heterogeneity. We can see that better growth shocks lead to better price schedules, as do worse outside option shocks to default. The magnitude of the former for the same size shock is much larger than the magnitude of the latter. What is interesting about the unobservable shock, though, is that it is state-contingent. For low debt levels, investors do not pay attention to unobservables and thus do not react. Thus, there is no difference across the schedules for debt-to-GDP levels lower than about 10% or so. As debt levels rise, though, they begin to pay attention and the two schedules begin to diverge.

The equilibrium policy functions are given in Figure 4. We can see that they share the common feature with the quantitative literature that better growth shocks lead to both higher debt levels and faster convergence to those debt levels.

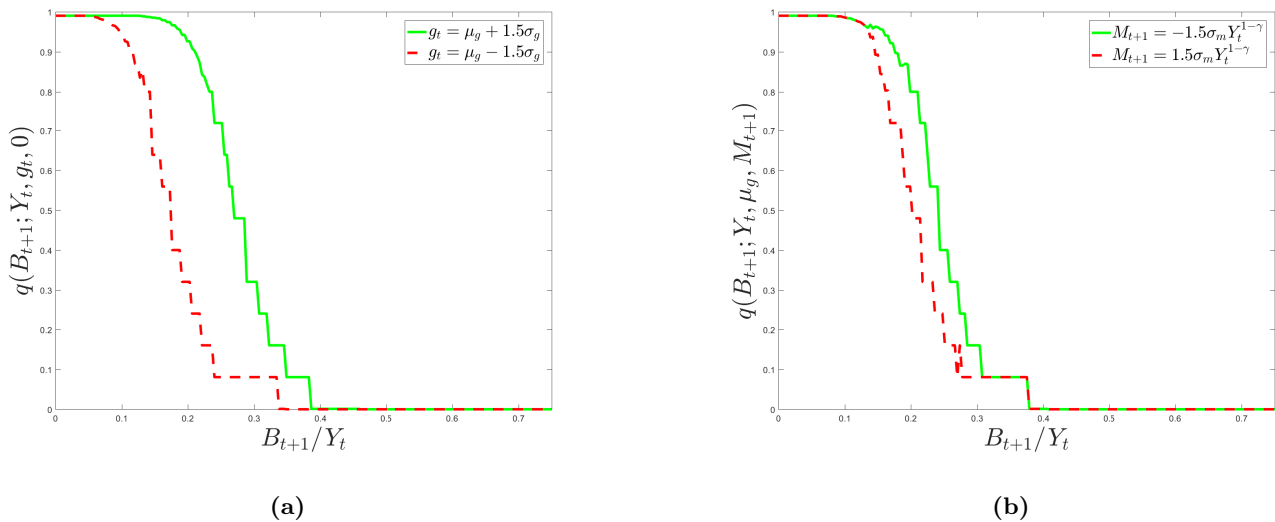


Figure 3: Equilibrium Demand Functions

Further, Figure 5 provides an event study surrounding a default. Again, the model behaves as a standard model would. We can see that spreads gradually increase prior to a default and exhibit a sharp spike the period before; growth is near or above trend leading up to default, deteriorating gradually at first but then exhibiting a sharp and unexpected drop just prior to the default.

What's novel in this model is the state-contingent acquisition of information: Investors optimally choose to acquire more information during times of crises i.e. near default. Recall that the ergodic mean of the debt-to-GDP ratio is 12.0%. We can see in the policy functions in Figure 6 that for debt levels below this investor attention depends on the underlying growth shock i.e. the observable shock. If growth is high and times are good, investors do not pay attention unless debt levels become very large. However, when growth is low, investors pay much attention for levels of debt even well below the ergodic mean. Notice further that for very high debt levels investors cease to pay attention as well: This is because default is near certain in these regions, regardless of the realization of the unobservables. Consequently, there is no point paying a cost to learn about the unobservable shocks.

We can see that attention increases during crises in Figure 7, which shows both the signal structure and acquired information leading up to an instance of default. We can see that signal precision and acquired information are relatively low 9 quarters prior to the event i.e. during non-crisis times. As publicly observable states signal that default is on the horizon, however, investors become more aggressive in their

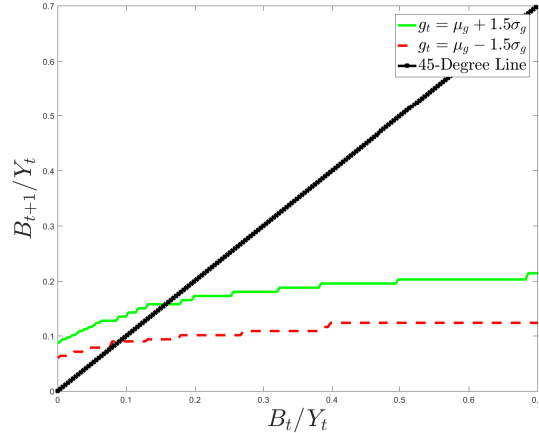


Figure 4: Equilibrium Policy Functions

acquisition of information, increasing it by nearly an order of magnitude (when measured in bits) on the cusp of a default event.

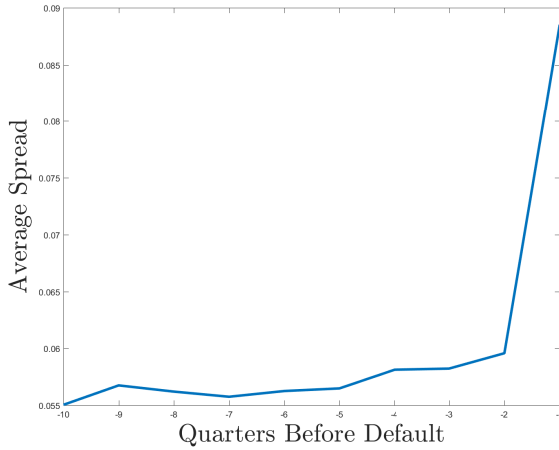
Figure 7 highlights clearly the state-contingent impact of costly information. It will lead us naturally to its implications for the pricing of sovereign risk.

3.3. Novel Implications

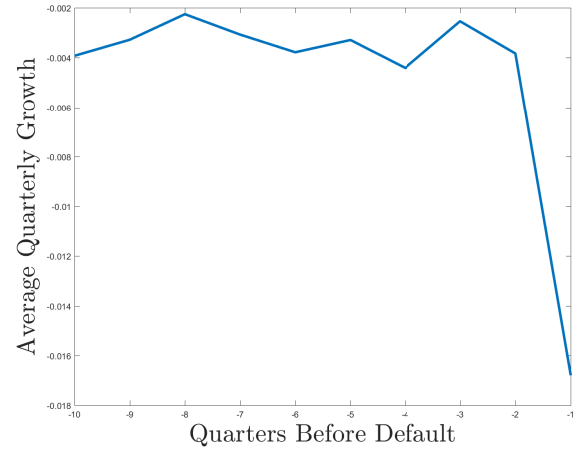
The state-contingent acquisition of information has strong effects on some model objects and negligible effects on others. This is shown in Figure 8. We can see that mean debt levels, spreads, and default frequencies are affected only negligibly by the level of information costs. Even though we allow for general equilibrium effects, it seems they are either not present or not strong enough to influence the first moments implied by the model.

But while first moments are relatively unaffected, both higher moments and marginal values of variables are substantially affected by information costs. Figure 8 also shows that the fraction of time investors spend paying attention to the sovereign and the spread volatility vary significantly with information costs. The intuition for the former is trivial; the intuition for the latter is that cheaper information means that unobservables get priced more often instead of being assumed at their average. This mechanically increases spread volatility.

The skewness of the spreads is also effected substantially, as can be seen in Figure 9. The effect is even



(a)



(b)

Figure 5: Benchmark Behavior Around Default

stronger on this moment than it is on the volatility. This is because exceedingly large spreads are almost always a result of unobservable shocks being both adverse and priced. The more often these shocks are priced, the more often investors are likely to capture instances in which they are adverse.

3.3.1. Time-Varying Volatility

First, spreads will exhibit increased volatility during times of crises, since lenders will price unobservables more accurately, rather than assuming them to be at their mean, which they do during non-crisis times to save on information costs. Further, the time-varying volatility will be strongly countercyclical and positively correlated with high spreads. This implies that one could interpret our framework as a microfoundation for such models as [Melino and Turnbull \(1990\)](#) or [Fernández-Villaverde et al. \(2011\)](#).

One would be right question, however, how much of this time-varying volatility is actually due to the information frictions. Part of the increase in the volatility prior to a default event comes mechanically from the fact that spreads will be significantly higher prior to a default. To address this question, we propose an alternative, model-free metric of time-varying volatility, which we call the **Crisis Volatility Ratio** or CVR. We define the CVR as follows: In a series of data, either simulated or empirical, let \hat{T} denote the set of all periods in which the change in the spread from the prior period is above the 95%ile of its distribution. We will call such events ‘crises,’ and there will mechanically be many more of these than

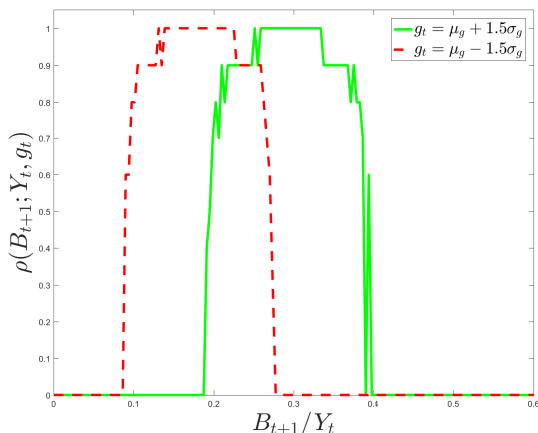


Figure 6: Information Acquisition Policy Function

actual defaults, which are $20\times$ less likely. With this notation, we can define the CVR as

$$CVR = \frac{1}{|\hat{T}|} \sum_{t \in \hat{T}} \frac{\hat{\sigma}_{t:t+5}}{\hat{\sigma}_{t-6:t-1}}$$

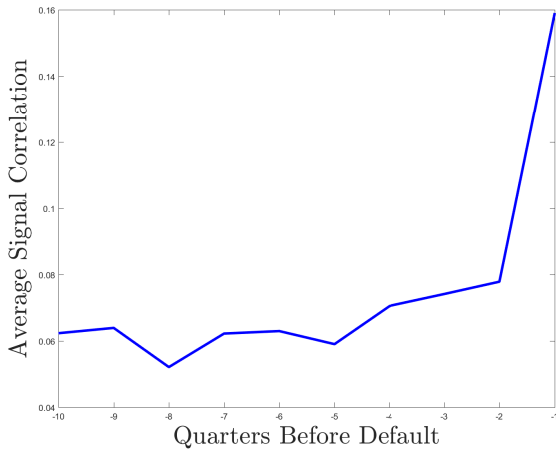
where $\hat{\sigma}_{x:y}$ is the sample standard deviation calculated using the periods from x to y . This metric is model-free and is a nice measure of stochastic volatility. If it is larger than one, then crisis periods tend to be more volatile than non-crisis periods.

We compute the CVR for the data and two different model scenarios: The benchmark model and the benchmark model with infinitely costly information. The results are given in Table 2. We can see that time-varying volatility is a strong feature of the data; much stonger than a standard model without information frictions would predict.⁸ Our model generates a 29% increase in the CVR, which brings it closer to the data.

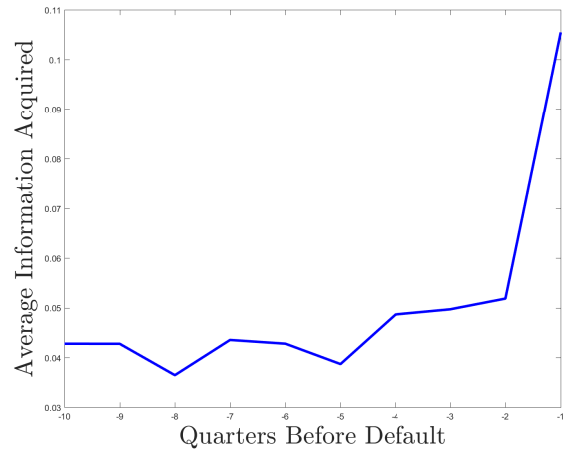
Table 2: Crisis Volatility Ratios

Data (Ukraine)	Benchmark Model	No-Information Counterfactual
3.67	2.12	1.68

⁸The no-information counterfactual here is the CVR produced by the following exercise: Hold fixed the equilibrium policy and default functions i.e. the risk to the lenders, and re-solve the lenders' problem assuming infinitely costly information. This provides an alternative, non-equilibrium price schedule that describes how a lender with no access to unobservable information would price the default risk. We compute an alternative sequence of spreads in the simulated data with this pricing schedule and use that to compute the counterfactual CVR.



(a)



(b)

Figure 7: Information Acquisition Before Default

While our calibrated model puts the number at 29%, comparative statics suggest that it could be much larger. Figure 10 gives the percent increase from the same counterfactual for a wide range of different information costs. It makes clear that the degree of time-varying volatility is monotone decreasing the information costs despite the potential for general equilibrium effects, and that this metric is *highly sensitive* to the information cost. Moving from the benchmark to slightly lower levels of information costs bring this percent increase from 29% to near 50% and then to near 100% in the full-information model.

It is worth noting that there is some time-varying volatility in the model even without information frictions. This is simply because default risk is asymmetric. Fluctuations in *observable* shocks are not always relevant for default risk and consequently are not always priced. This implies lower price volatility in normal times than crises. This is true in any model of endogenous sovereign default and, to our knowledge, is a previous undocumented finding. Nevertheless, our model’s ability to **amplify** this underlying time-varying volatility is substantial and noteworthy.

The variation in the CVR across information costs implies an interesting and testable hypothesis. Since the IT revolution, it is a relatively undebatable point that information costs have been falling over the past 30 years or so. Our model suggests that this should imply more time-variation in spread volatility. We test this prediction in the EMBI data and find it to be true. In particular, we compute the percentage increases in the CVRs for every country in the EMBI database between the time periods 1994-2003 and

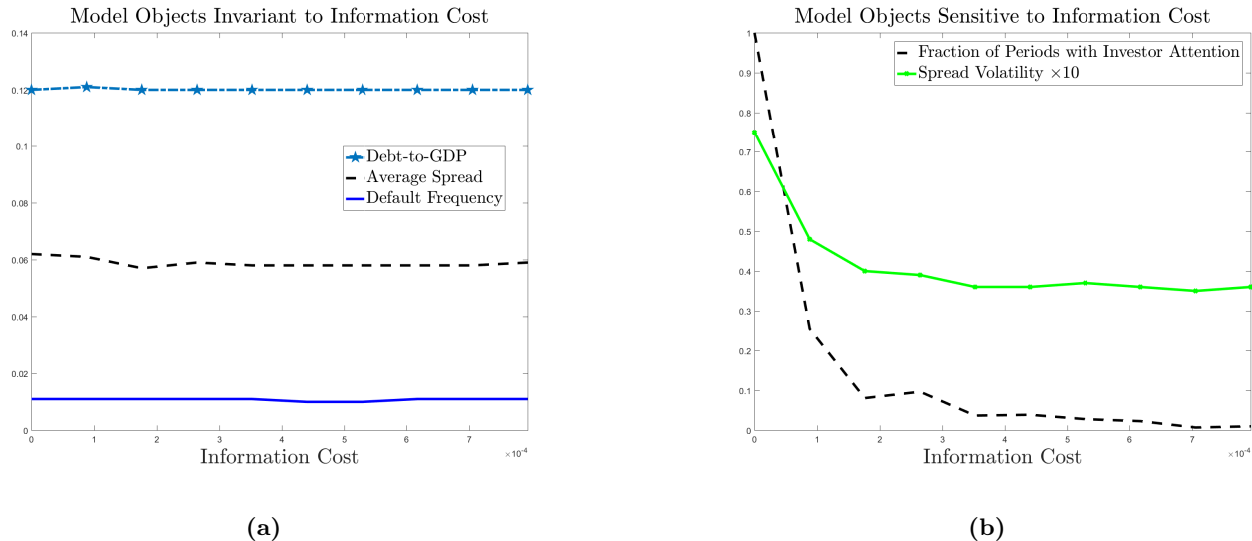


Figure 8: Model Moments Across κ

2004-2014. We find the average of these percentage increases to be 192.7%, which is surprisingly large but not inconsistent with the model’s prediction, which suggests that the CVR could more than double with relatively small changes in information costs.

These results are not driven by the financial crisis that affected most of the global economy in 2008. While it’s true that volatility increased in many developed nations, the country-risk spread in emerging markets actually saw a substantial *decrease* in spread volatility over these times periods, with the average percentage change being -14.5% . This could be for many different reasons, from successful implementation of reforms to unconventional monetary policy in developed economies. We do not need to take a stance on why exactly, but what is noteworthy is that while overall volatility has fallen over this time period, it has become much, much more time-varying. This suggests strongly that our mechanism is at work in the data not just in Ukraine, but in many different countries. In fact, the only countries in the whole dataset of 17 countries to exhibit a falling CVR are Colombia and Brazil.

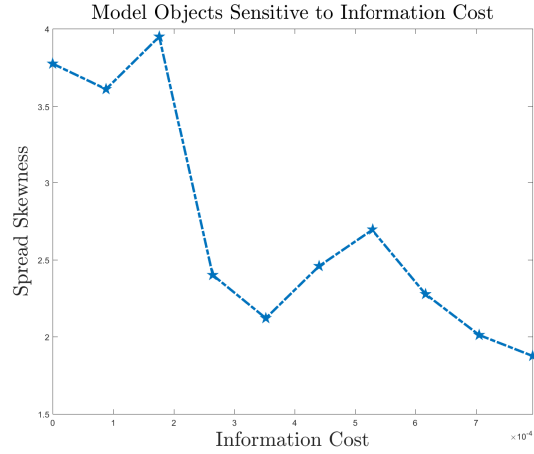


Figure 9: Skewness Across κ

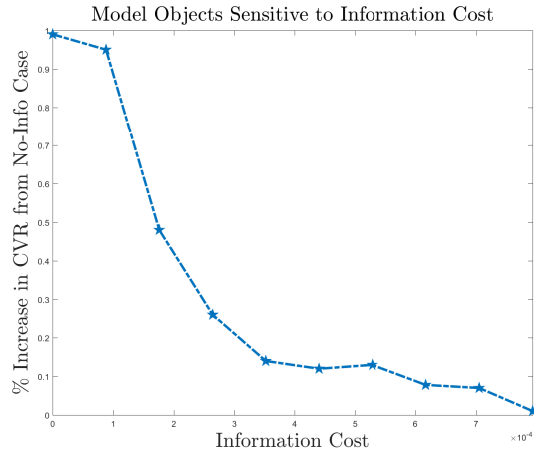


Figure 10: Crisis Volatility Ratios Across κ

3.3.2. Default-Risk Inference

Second, the risk-decomposition of spreads is *not* the same during crisis times and non-crisis times. To understand this, first note that we can intuitively break the spread on sovereign debt into four categories:

$$\begin{aligned}
 Spread_t = & \text{ObservableDefaultRisk}_t + \text{UnobservableDefaultRisk}_t + \\
 & \text{ObservableRiskPremium}_t + \text{UnobservableRiskPremium}_t
 \end{aligned}$$

The fourth component will vary across states depending on the lenders' endogenous information set. When lenders pay the cost to observe normally unobservable states, they learn more about the realization of those unobservable states. Hence they can better tolerate that risk and the unobservable risk premium decreases.⁹

What does this imply for default risk inference? Consider an observable crisis e.g. a large negative growth shock for a highly indebted sovereign. These are exactly the sort of crises considered by the vast majority of the literature. While default *risk* is higher during such a crisis, so too is *tolerance for that risk*, since investors are paying more attention and acquiring better signals about the unobservables. If the crisis is only driven by observable shocks and unobservable shocks are truly playing no role, then the risk premium coming from unobservables will be lower while unobservable default risk does not change.

This implies that the unobservable risk premium comprises a *relatively smaller component of the spread during an observable crisis*. Consequently, if an econometrician were not to take this into account, instead employing a standard model with constant risk tolerance, she would tend to *underestimate* default probabilities during crises: She would assume the risk spread to be higher than it actually is, and consequently a default risk which is lower than it actually is for a fixed spread.

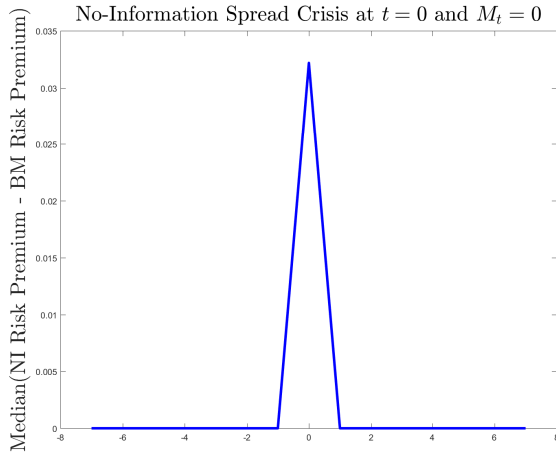
Our model allows us to quantify this effect. In particular, we construct an artificial, non-equilibrium no-information price schedule as we did in the previous subsection. This can be seen in Figure 11.

What is perhaps most interesting about these consequences of costly information acquisition is that they are *country-specific*. Thus, they cannot be controlled for using *global* metrics, such as the CBOE VIX or the P/E ratio, as is often done (Aguiar et al. [2016] or Bocola and Dovis [2015]). Rather, our theory suggests that in order to accurately assess default risk, some metric of investor attention, such as SVI, must be acquired and controlled for.

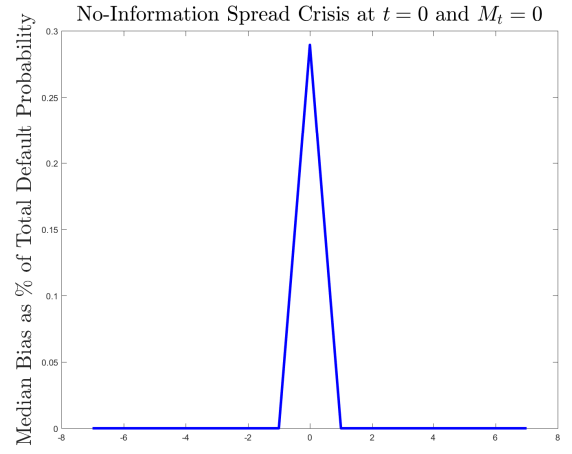
3.3.3. Transparency

Costly information acquisition can be interpreted in many ways in the context of our model. Up to this point, we have focused on the interpretation that it is difficult to lenders to acquire information about a sovereign. However, information costs in our framework could also be understood as scaling the level of transparency that a sovereign has about its own domestic affairs and finances.

⁹Note that this says nothing about what happens to the level of the spread, which could increase or decrease as information is acquired since the default probability could scale up or down as new information is acquired.



(a) Risk Premium Difference: Level



(b) Risk Premium Difference: Fraction of Underlying Default Risk

Figure 11: Potential Bias in Default Probability Inference

If we interpret κ as a measure of transparency, then our model has some interesting consequences. Policymakers generally argue that improved transparency is a desirable policy goal. Our model suggests that this is not necessarily true; the sovereign would prefer to be neither fully transparent nor completely opaque regarding unobservable variables. This can be seen in Figure 12, which plots sovereign welfare as a function of information costs.

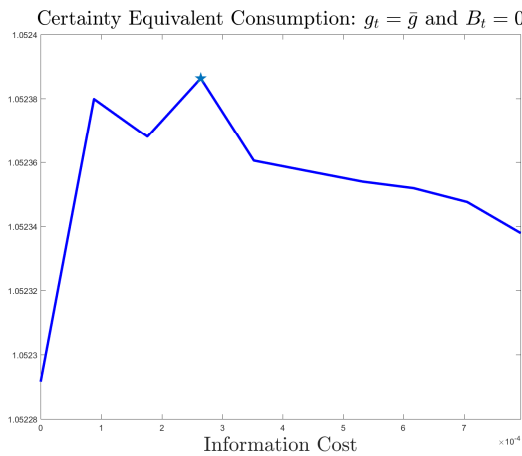


Figure 12: Sovereign Welfare Across Information Costs

From the figure, we can deduce that the sovereign prefers intermediate levels of information costs to either extreme. This is because there is a tradeoff associated with transparency. When information is

infinitely costly, lenders always demand a risk premium for the unobservables, particularly during crisis times. This makes it more expensive for the sovereign to borrow and service debt precisely when his marginal utility is the highest i.e. during crises. On the other hand, when the sovereign is fully transparent, this crisis risk premium disappears but it is replaced by substantial price volatility, since unobservable shocks are now constantly priced. The risk-averse sovereign does not like this either.

Consequently, the model suggests that the optimal level of transparency is somewhere in the middle, where payoff-relevant information is accessible to investors, but somewhat costly to acquire. In fact, the model suggests that the benchmark calibration implies a level of information costs that is close to welfare-maximizing.

4. Conclusion

In this paper, we explored the consequences of costly information acquisition on the pricing of sovereign risk. We constructed a structural model of endogenous default and information acquisition and calibrated that model to match data from EMBI, the World Bank, and Google SVI.

We demonstrated that costly information acquisition can generate time-varying volatility in the country risk spread; implies a misspecification bias of default probabilities in standard econometric inference of default probabilities from yield data; and generates non-trivial welfare results with regard to transparency.

Possible extensions to our framework could include rollover crises in the vein of [Cole and Kehoe \(1996\)](#), long-maturity debt ([Hatchondo and Martinez \[2009\]](#) or [Chatterjee and Eyigungor \[2012\]](#)), or persistent unobservable processes. The intuition of our results would not change with any of these extensions but the quantitative results may be affected.

This paper lays the groundwork for more careful econometric work in the identification of sovereign risk. In particular, it provides a way to account for the negative bias that results in default probability estimates during observable crises.

References

- Aguiar, Mark and Gita Gopinath**, “Defaultable Debt, Interest Rates, and the Current Account,” *Journal of International Economics*, 2006, 69 (1), 64–83.
- , **Satyajit Chatterjee, Harold Cole, and Zachary R. Stangebye**, “Quantitative Models of Sovereign Debt Crises,” *Handbook of Macroeconomics, Volume II*, 2016.
- Arellano, Cristina**, “Default Risk and Income Fluctuations in Emerging Economies,” *American Economic Review*, 2008, 98 (3), 690–712.
- Barber, Brad M. and Terrance Odean**, “All That Glitters: The Effect of Attention and News on the Buying Behavior of Individual and Institutional Investors,” *Review of Financial Studies*, 2008, 21 (2), 785–818.
- Bi, Huixin and Nora Traum**, “Estimating Sovereign Default Risk,” *American Economic Review*, 2012, 102 (3), 161–166.
- Bocola, Luigi**, “The Pass-Through of Sovereign Risk,” *Job Market Paper*, 2014.
- and **Alessandro Dovis**, “Self-Fulfilling Debt Crises: A Quantitative Assessment,” *Working Paper*, 2015.
- Chatterjee, Satyajit and Burcu Eyigungor**, “Maturity, Indebtedness, and Default Risk,” *American Economic Review*, 2012, 102 (6), 2674–2699.
- Cole, Harold L. and Timothy J. Kehoe**, “A Self-Fulfilling Model of Mexico’s 1994-1995 Debt Crisis,” *Journal of International Economics*, 1996, 41 (3-4), 309–330.
- Da, Zhi, Joseph Engelberg, and Pengjie Gao**, “In Search of Attention,” *Journal of Finance*, 2011, 66 (5), 1461–1499.
- Eaton, Jonathon and Mark Gersovitz**, “Debt with Potential Repudiation: Theoretical and Empirical Analysis,” *Review of Economic Studies*, 1981, 48 (2), 289–309.
- Fernández-Villaverde, Jesús, Pablo A. Guerron-Quintana, Juan Rubio-Ramirez, and Martin Uribe**, “Risk Matters: The Real Effect of Volatility Shocks,” *American Economic Review*, 2011, 101 (6), 2530–2561.

- Gabaix, Xavier**, “A Sparsity-Based Model of Bounded Rationality,” *Quarterly Journal of Economics*, 2014, *129* (4), 1661–1710.
- Gervais, Simon, Ron Kaniel, and Dan H. Mingelgrin**, “The High-Volume Return Premium,” *The Journal of Finance*, 2001, *56* (3), 877–919.
- Hatchondo, Juan Carlos**, “Asymmetric Information and the Lack of International Portfolio Diversification,” *Working Paper University of Rochester*, 2004.
- and **Leonardo Martinez**, “Long-Duration Bonds and Sovereign Default,” *Journal of International Economics*, 2009, *79* (1), 117–125.
- Lizarazo, S.V.**, “Default Risk and Risk Averse International Investors,” *Journal of International Economics*, 2013, *89*.
- Mackowiak, Bartosz and Mirko Wiederholt**, “Optimal Sticky Prices Under Rational Inattention,” *American Economic Review*, 2009, *99* (3), 769–803.
- Melino, Angelo and Stuart M. Turnbull**, “Pricing Foreign Currency Options with Stochastic Volatility,” *Journal of Econometrics*, 1990, *45* (1-2), 239–265.
- Mendoza, Enrique G. and Vivian C. Yue**, “A General Equilibrium Model of Sovereign Default and Business Cycles,” *Quarterly Journal of Economics*, 2012, *127* (2), 889–946.
- Passadore, Juan and Juan Pablo Xandri**, “Robust Conditional Predictions in Dynamic Games: An Application to Sovereign Debt,” *Job Market Paper*, 2015.
- Peng, Lin and Wei Xiong**, “Investor Attention, Overconfidence, and Category Learning,” *Journal of Financial Economics*, 2006, *80* (3), 563–602.
- Seasholes, Mark S. and Guojun Wu**, “Predictable Behavior, Profits, and Attention,” *Journal of Empirical Finance*, 2007, *14* (5), 590–610.
- Shannon, Claude E.**, “Channels with Side Information at the Transmitter,” *IBM Journal of Research and Development*, 1958, *2* (4), 289–293.

Sims, Christopher A., “The Implications of Rational Inattention,” *Journal of Monetary Economics*, 2003, 50 (3), 665–690.

—, “Rational Inattention: Beyond the Linear-Quadratic Case,” *American Economic Review*, 2006, 96 (2), 158–163.

Stangebye, Zachary R., “Dynamic Panics: Theory and Application to the Eurozone,” *MPRA Working Paper No. 69967*, 2015.

Van Nieuwerburgh, Stijn and Laura Veldkamp, “Information Immobility and the Home Bias Puzzle,” *Journal of Finance*, 2009, 64 (3), 1187–1215.