Endogenous formation of coops and cooperative leagues

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Abstract

The labor-managed Mondragon cooperatives in the Basque country, and La Lega coops concentrated in North Central Italy, are grouped into leagues that enable them to reap economies of scale in key services such as R&D, marketing and finance. These leagues are relatively rare and there are fewer than a dozen of them globally. We develop a game-theoretic model of league formation to capture some of the strategic incentives behind the formation of labor-managed cooperatives (coops) and their agglomeration into a league. We then compare these incentives with those of conventional profit-maximizing firms to organize into a league. The main result of this paper shows that a divergence in these incentives stemming from their organizational differences may lead to the formation of a league of firms but not one of coops. This turns out to be true even though the coop has lower costs of production and the existence of a coop league would have been socially efficient. Anticipating the non-existence of a coop league then creates a disincentive for individual agents to form coops in the first place. This explains the relative rarity of coops, competing individually or as a part of a league, with conventional firms in imperfect markets.
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1. Introduction

In the Mondragon cooperatives in the Basque country, and La Lega coops concentrated in North Central Italy, tens of thousands of workers operate self-managed industrial cooperatives (or *coops*), which in turn are grouped together in leagues that enable them to reap economies of scale in key services such as R&D, marketing and finance.[1] Coop leagues have also emerged in developing countries such as India (Isaac et al., 1998; Gulati et al., 2002). However, these leagues are relatively rare and there are fewer than a dozen of them globally. This stands in marked contrast to the large number of collaborative leagues among conventional profit-maximizing firms (or *firms*). Our paper is an attempt to understand, within a general game-theoretic model of coalition formation, the strategic incentives of individual agents to form coops and for these coops to organize themselves into a league. We then examine how differences in the organizational structure of coops cause their incentives to differ from those of firms to similarly agglomerate into a league. In particular, we argue that coops have lower marginal costs of production than firms due to product development but higher costs of league formation, where leagues offer another avenue to coops and firms for reducing

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[1] For more on labor-managed coops, please see Bonin et al. (1993), Dow (1993), Martin (2000), Vanek (1970) and Ward (1958).

their marginal costs of production. The main result of this paper shows that a divergence in these incentives stemming from their organizational differences may lead to the formation of a league of firms but not one of coops. This turns out to be true even though coops have a lower marginal cost of production and the existence of a coop league would have been socially efficient. Anticipating the non-existence of a coop league then creates a disincentive for individual agents to form coops in the first place. Our paper therefore draws upon the theory of non-cooperative coalition formation games to explain the relative rarity of coops, competing individually or as a part of a league, with conventional firms in imperfect markets.

We construct a three-stage model of group formation comprising of both coops and firms. All firms are ex ante identical and their number is exogenously fixed. In the first stage, ex ante symmetric agents decide whether to supply labor to the firm (for some exogenously given wage) or organize themselves into a coop. As a coop, the members compete in the product market with the firms (and other coops) and receive an equal share in their coop's profits. Forming a coop has an organizational comparative advantage in terms of peer monitoring of labor effort, peer monitoring of quality, and the adoption of process innovations. These comparative advantages translate into lower marginal costs of production for coops relative to firms. However there are limits to the cost reduction that can be effected in this manner. Adding new members also reduces each member's share of the profits. Moreover, forming a coop imposes costs in the form of coordinating the decisions and activities of the participating agents. Forward-looking agents take these benefits and costs into account when making a decision to form coops. Entry into a coop is generally restricted. We therefore model coop formation as an *exclusive membership* game due to Hart and Kurz (1983) according to which a coop is formed if and only if there is complete unanimity among member agents with regard to its composition.

At the start of the second stage, the number of coops (and firms) are given. From now on we will follow the convention of referring to both coops and firms as *players* when discussing aspects of the model that are common to both. We will explicitly refer to either only when they are treated differently in the model. Individuals who make up a coop are called *agents* as distinct from players. In the second stage, players have to decide whether or not to participate in a league. Coops can form a league only with other coops; similarly firms can only agglomerate with other firms. This is because leagues provide services that are frequently specialized according to the needs of the organizational forms of their members. For example, coops present special requirements for legal and tax advice, insurance and financial services. Thus each type of player benefits most from a specialized league of their own.

The details of the second-stage league formation game are dictated by three key features that are observed in both Mondragon and La Lega. First, leagues generally have open admission policies. Second, leagues provide services that reduce costs of production for members. Third, each member has to bear some cost of league participation. Table 1 offers a summary of these features.[2]

Even though these characteristics may not always apply to firms, we assume that firms operate under the same set of rules in order to meaningfully compare their equilibrium behavior with those of coops within the same strategic environment.

Since leagues have open admission policies, we model them using the *cartel formation* game of D'Aspremont et al. (1983). All players choose a message from the set $M = \{Yes, No\}$. All coops announcing *Yes* belong to the league while those announcing *No* remain singletons; the same is true of firms. Membership is open because players can join their respective leagues by announcing *Yes*. The resulting league is *internally stable* if no member has an incentive to exit, and it is *externally stable* if no outside player has an incentive to enter the league. Only one league is allowed to form. We characterize the size of the coop league that is both internally and externally stable and contrast it with that of firms.

The existence and activities of a coop league can lower cost of production for member coops through a variety of channels: lowered input costs through buying and negotiating power, shared costs of relevant innovation, technological progress and quality upgrading, improved financial intermediation, reduced risks through league explicit and implicit insurance, coordinated marketing strategies, development of relevant professional services, and lower internal costs of bargaining among members. Many of these channels are also present in a league of firms. The trade-off from joining a league is the membership cost incurred by the participating players in coordinating their activities as well as paying for the services provided by the leagues. It is with regard to the costs of league formation that the next difference in

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[2] More institutional details of Mondragon and La Lega are available in Ammirato (1996), Clamp (2000), Smith (1994), Smith (2003) and Whyte and Whyte (1991).

Table 1
Comparing Mondragon and La Lega on three dimensions emphasized in the model

League/theme	Mondragon, Basque Country, Spain	La Lega (Legacoop), Italy
Brief overview	Network of about 82 mid-sized industrial and service coops in Basque Country of Spain, with a total workforce of about 68,000	Network of over 5000 worker coops in Italy, plus parallel networks of consumer, housing and other specialized coops
Open admissions policy	Coops can exit and enter MCC (e.g. seven-member Ulma group left and subsequently rejoined). Entry is free in principle, but direct competitors in an industry may be expected to participate in joint venture activities	Any coop that adheres to basic principles and pays league dues is free to join (or to leave) at any time, regardless of sector or degree of market competition with other member coops
Cost reduction activities	Common administrative functions, including corporate strategy, are carried centrally, directed by a board comprised of presidents of member coops. Some MCC coops provide specialized services for members, including innovation, financial, export, and legal services. Separate consortia handle social insurance, including pensions, unemployment, and health care	Legacoop sponsors over 30 specialized institutes and consortia providing services to coop members, including joint purchasing, innovation, PR, organizational development, training, publishing, inter-coop subcontracting, exporting, and financial services. Takes advantage of returns to scale, and creates a market for specialized services relevant to coop features
League costs	Members contribute a share of value added for administrative services. Social insurance is handled by central institutions. Risk-sharing is carried out through contributions of a share of net income, and can be used for short-term cross-subsidies	Member coops pay league dues, and consortia and institutes receive small commissions per transaction

the two organizational forms comes into play. In contrast to firms, coops present special requirements for legal advice, industrial relations, organizational development, tax advice, specialized insurance, marketing strategies and financial services. A league of firms can simply draw on the existing structures and institutions to reduce their cost of production. A league of coops, on the other hand, has to invest considerably more in creating the framework and institutions suited to their relatively unique organizational form in order to effect the same cost reduction. This increases the cost of league formation for coops relative to firms.

Another factor that may raise the cost of league formation for coops is the number of decision-makers that are involved. Negotiating, implementing and administering the various cost-reducing activities of the league requires coordination among the decision-makers in a league. A firm only has a *single* decision-maker (owner) while a coop comprises of *multiple* decision-makers (worker–owners). In a league of size k, a decision-maker in a firm has to coordinate with $k - 1$ other decision-makers. In a coop league of size k, where each coop has $h > 1$ agents, a decision-maker has to coordinate with $(h - 1) + h(k - 1)$ other decision-makers. Coops can of course reduce these coordination costs by delegating one agent to represent each coop. However, the act of delegating the managerial function to one member agent and setting up a mechanism for the flow of information between the manager and the other agents in the coop is still likely to raise the coordination costs for coops above those for corresponding firms.

In the third and final stage, all players, those in a league as well as those outside, compete as Cournot oligopolists in a homogeneous product market. We solve for a subgame perfect equilibrium. Given the coop formation from the first stage, we start by considering the incentives of both sets of players to form a league in the second stage. Our first set of results are for a general specification of league costs. We start by showing that there is an endogenously determined threshold of members that must be reached before a league becomes viable. Further this threshold is *smaller* for firms than for coops. The difference in thresholds is explained by the fact that while firms may have higher marginal costs of production than coops, they have lower costs of league formation.

Our paper then identifies an additional reciprocal *negative externality* that each league imposes on the other: the formation and expansion of one league, say that of firms, lowers the incentives for the formation of the other league (for e.g. the coop league). This is simply a reflection of the Cournot equilibrium property that the reduction in the marginal costs of a subset of players has an adverse effect on the output and profits of the remaining players. Coupled with

the fact that firms have a lower threshold for league formation, this negative externality indicates why coop leagues may not come into existence, or remain small, in industries where firms are already organized into leagues. We then backwardly induct to the first stage and show how the existence of coops is tied to the existence of a coop league. In particular, we show that the non-existence, or small size, of a coop league depresses the incentive of individual agents to form coops. Anticipating correctly in equilibrium that their Cournot profit share will be smaller than the exogenous wage means that agents do not organize into coops at all.[3]

We then consider the issue of efficiency. An efficient league structure is one that maximizes the sum of consumer surplus and aggregate profits of coops and firms (net of any costs of group formation). We show that for sufficiently low cost of group formation, it is efficient for all coops to participate in a league and all firms to form a league as well. We also show that there is in general a tension between the equilibrium and efficient outcomes. In particular, we demonstrate that there could be a range of costs of league formation where efficiency dictates that only a coop league should form and all firms should remain singletons; however, over the same range of costs, the equilibrium league structure is the exact opposite where all firms are organized into a league and all coops remain singletons or do not come into existence at all.

Our paper is a contribution to the growing literature on the endogenous formation of coalitions.[4] Our first contribution lies in considering the interaction between two asymmetric groups of players and examining the conditions under which one group may not form at all even though efficiency would dictate that they form and agglomerate into a league. It therefore provides a simple framework to study the incentives of heterogeneous players to organize into leagues under different organizational forms. To the best of our knowledge, it is the first paper that attempts to explain the rarity of coops and coop leagues by adapting the theory of endogenous coalition formation. Our second contribution is that we point to a channel other than coordination failures to explain the rarity of coop leagues. In models of endogenous coalition formation where players move simultaneously, one of the Nash equilibria is usually the singleton coalition structure because players cannot *unilaterally* force the formation of a non-trivial league. Forming a stable league in the second stage requires the threshold number of players to announce *Yes*; otherwise, it is always a mutual best response for players to announce *No*. Therefore it is sometimes possible to refine the set of Nash equilibria by considering coalitional deviations and restricting attention to coalition-proof Nash equilibria (for example, Bloch, 1997). These coordination problems also constitute a problem in our model. However, the presence of a mechanism that allows players to coordinate mutually profitable coalitional deviations may still not be enough to guarantee that a non-singleton coop league will emerge. If a coordinating mechanism is indeed available, then it should be available to *both* firms and coops. There is no apparent reason why any one set of players should be precluded from such a mechanism if it exists. But then firms, because of their lower league threshold, would be able to coordinate and form a league before coops. The negative externalities resulting from the league of firms may then be strong enough to prevent the formation of a coop league. This in turn may lead to no coops being formed at all. Thus non-existence of coops may continue to be the equilibrium in a model that permits coalitional deviations.[5]

The paper is organized as follows. The basic model is described in Section 2. Section 3 develops some of the mathematical tools that are helpful in characterizing the leagues formed by coops and firms. Section 4 offers a characterization of the equilibrium of the league formation game. Section 5 considers the coop formation game. Section 6 offers a comparison between equilibrium and efficient league structures. Section 7 concludes by pointing out avenues for further research. All proofs are collected in Appendix B.

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[3] This argument also brings to the fore an interesting average-marginal distinction. Since a firm is a single decision-maker, it compares the *marginal* benefit (change in its profits when the league expands by one more decision-maker) to the cost of joining a league. A coop on the other hand has multiple decision-makers with an equal share in any increment in profits when the league expands. Therefore, when deciding to form a coop that will be part of a league, a coop member compares the *average* benefit (the change in profits when the league expands by one more coop *divided* by the number of decision-makers) to the cost of group formation (his share of the league costs plus the cost of organizing the coop). To the extent that average benefits are less than marginal benefits, coops may not emerge even if group formation costs were the same for coop members and firms.

[4] Please see for example Bloch (1995), Bloch (1997), D'Aspremont et al. (1983), Goyal and Joshi (2003), Hart and Kurz (1983), Yi and Shin (2000) and Yi (1998).

[5] The notion of coalitional deviations and negative externalities may suggest that firms behave pre-emptively to block the formation of a coop league. However, as noted by a referee, a league of firms may also exist due to historical reasons and not deliberate pre-emption. The final result regarding non-existence of a coop league would continue to be valid (due to the negative externalities exerted by the league of firms).

2. The model

In stage 1 there is a fixed number of m identical firms and a finite set $\mathcal{N}$ of identical agents who can organize into coops or work for the firms at a given wage w. We model coop formation as an exclusive membership game (see Hart and Kurz, 1983). Let $\mathcal{M}$ denote a message set containing all possible subsets of $\mathcal{N}$ and $\omega \in \mathcal{M}$ denote a message. The strategy s_i for agent i is a message announcing a set of agents with whom i would like to form a coalition. All agents simultaneously choose their strategy, i.e. the coalition of agents to which they wish to belong. A coalition is formed if and only if there is complete unanimity among these agents with regard to its composition. Formally, agents who have announced the same message ω form the coalition $C(\omega) \equiv \omega$ if and only if for each $i \in \omega$, $s_i = \omega$. A coalition breaks down into singletons if one of the agents in the coalition changes his message.

Let us suppose that n coops are formed in the first stage. The n coops and m firms compete as Cournot oligopolists in a homogeneous product market with linear inverse demand:

$$P = \alpha - \sum_{i=1}^{n} q_i - \sum_{j=1}^{m} q_j \tag{1}$$

Both firms and coops produce under constant returns to scale and thus have constant marginal costs. The advantage of forming a coop is that it enables members to have an equal share in the profits as well as reduce the marginal costs of production. The reduction in marginal cost occurs in the following manner. The possibility of peer monitoring of labor effort and quality, and incentives created by having a common stake in the success of the coop, usually implies that coop products are qualitatively different from those produced by conventional firms (see Smith, 1994). Borrowing an idea from Spence (1984, Footnote 2), we can interpret this qualitative difference as the coop product providing more services to the consumer and allow cost reduction to occur due to product development.[6]

There are also costs to coop formation, given by the real-valued function $v(h)$, where h denotes the number of agents in the coop. One large component of $v(h)$ is fixed. It involves the organizational expenditure in starting a coop and obtaining the required financial, organizational, legal and insurance services. Therefore having more members allows agents to lower the per member share of these organizational costs. There is also a variable component to the cost of coop formation since having more members increases the cost of coordinating the activities of a larger set of agents. Thus we can expect $v(h)/h$ to initially fall with h but increase after h exceeds some threshold and it becomes increasingly difficult to efficiently manage a large number of member agents. Our analysis allows the function v to be specified quite generally.

While forming a coop can reduce marginal costs of production due to product development, there are limits to the cost reduction that can be effected in this manner. Coops can only decrease their costs further by participating in a league in stage 2 and sharing the benefits of agglomeration. Let k denotes the number of coops in a league, the cost function of a member coop of producing output q is given by

$$C(q) = [\gamma_0 - \delta - \gamma(k-1)]q \tag{2}$$

The constant δ represents the cost reduction for a coop due to product development.[7] The term $\gamma(k-1)$ represents the additional linear cost reduction due to membership in a league of size k. If a coop is not a member of a league, then $k = 1$ for the coop. The marginal cost is independent of output and, following Bloch (1995), is linearly decreasing in the size of the league. The cost of production for a firm who is a member of a league of size j is given by:

$$\hat{C}(q) = [\gamma_0 - \gamma(j-1)]q \tag{3}$$

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[6] Formally, suppose consumers demand services, s, delivered through the product. Let $g(d)$ denotes the quantity of services per unit of the product, where d is a "quality" parameter and $g'(d) > 0$. Then $s = g(d)q$ is the total quantity of services provided by a coop. Assuming a constant returns to scale cost function, $C(q) = cq$. Thus $C(s/g(d)) = c(s/g(d))$ and a coop can reduce costs through product development, i.e. increasing d to provide more services per unit relative to firms.

[7] It can be argued that the cost reduction δ due to improved product quality from the formation of a coop will be a function of the number of members h in the coop, with some limit on the degree of cost reduction that can be engendered by improving product quality. This just complicates the computations and does not add anything of significance to the main result of the paper. Therefore, in the interests of tractability, we take the cost reduction δ from the formation of a coop to be independent of h.

The cost for a singleton firm is determined by setting $j = 1$. Note that for the same league size, coops have lower marginal cost than firms by the parameter δ. In the subsequent discussion we will maintain the parametric restriction $\alpha - \gamma_0 > \gamma m(m - 1) + \gamma|\mathcal{N}|(|\mathcal{N}| - 1)$, where $|\mathcal{N}|$ denotes the cardinality of the set of agents, in order to ensure that all players, whether they belong to their respective league or not, will produce a positive output in the Cournot equilibrium.

The cost to an individual firm of being a member in a league with $j - 1$ other firms is given by the function $c(j - 1)$, $j > 1$. The total cost to the league (aggregating over all member firms) is therefore $jc(j - 1)$. Recall that the cost of league formation is higher for coops than firms. Therefore we will assume that the cost to an individual coop of h agents from participating in a league with $k - 1$ other coops is given by $\mu(h)c(k - 1)$ where μ is continuously differentiable real-valued function satisfying $\mu(h) > 1$ for all $h \geq 1$ and μ is increasing in h. Thus costs of league formation are increasing in h since coordination costs are greater with more decision-makers being involved. The total cost to a coop with h members of first organizing a coop and then joining a league is therefore $v(h) + \mu(h)c(k - 1)$. Aggregating these costs over all member coops yields the cost to the coop league. We show that the main result of the paper can be obtained for a general specification of league costs, $c(k)$. In order to precisely characterize the equilibrium and examine its comparative-static properties, it will be convenient to work with the case where the league cost is a linear function of league size, i.e. $c(k - 1) = f(k - 1)$, $f > 0$.

We will distinguish between members of a league and non-members by using the superscripts M and NM, respectively. Since all agents are ex ante identical, in a subgame perfect equilibrium all coops that will subsequently join the league will have the same number of agents, h^M, while those who will remain outside the league will have the same number of agents, h^{NM}. Suppose that the coop league has k members while the league of firms has j members. The net profits of a member coop minus the membership costs of the league are

$$\Pi^M(k, j) - \mu(h^M)c(k - 1) = \frac{1}{(N + 1)^2}\left[(\alpha - \gamma_0 + (m + 1)\delta) + \gamma(k - 1)(N - k + 1) - \gamma j(j - 1)\right]^2$$
$$-\mu(h^M)c(k - 1) \tag{4}$$

where $N = m + n$. The net profits of a decision-maker in a non-member coop are simply its share of the Cournot profits since there are no membership costs:

$$\Pi^{NM}(k, j) = \frac{1}{(N + 1)^2}\left[(\alpha - \gamma_0 + (m + 1)\delta) - \gamma k(k - 1) - \gamma j(j - 1)\right]^2 \tag{5}$$

Note that member coops earn larger gross profits (before subtracting membership costs) than non-member coops by virtue of having lower marginal costs of production. In a Cournot equilibrium, players with lower marginal cost have greater output and profits.

The *internal stability* (IS) condition for the league requires that no member of the league should be willing to withdraw from the league. In other words, the net profits of each member coop of a league of size k should be at least as great as their net profits if they decide to withdraw and reduce the league size to $k - 1$:

$$\frac{\Pi^M(k, j)}{\mu(h^M)} - c(k - 1) \geq \frac{\Pi^{NM}(k - 1, j)}{\mu(h^M)} \tag{6}$$

Similarly, the *external stability* (ES) condition for a league requires that non-member coops should not get a higher net profit from joining the league and thereby expanding its size to $k + 1$ members:

$$\frac{\Pi^M(k + 1, j)}{\mu(h^M)} - c(k) \leq \frac{\Pi^{NM}(k, j)}{\mu(h^M)} \tag{7}$$

A coop league of size k is *stable*, given j, if it satisfies both the IS and ES conditions.

We now turn to firms whose Cournot payoffs for members of the league are

$$\hat{\Pi}^M(k, j) - c(j - 1) = \frac{1}{(N + 1)^2}[(\alpha - \gamma_0 - n\delta) + \gamma(j - 1)(N - j + 1) - \gamma k(k - 1)]^2 - c(j - 1) \tag{8}$$

For non-member firms:

$$\hat{\Pi}^{NM}(k, j) = \frac{1}{(N + 1)^2}\left[(\alpha - \gamma_0 - n\delta) - \gamma j(j - 1) - \gamma k(k - 1)\right]^2 \tag{9}$$

A league of firms is, respectively, internally and externally stable if:

$$\hat{\Pi}^M(k, j) - c(j - 1) \geq \hat{\Pi}^{NM}(k, j - 1), \qquad \hat{\Pi}^M(k, j + 1) - c(j) \leq \hat{\Pi}^{NM}(k, j) \tag{10}$$

A league of firms of size j is *stable*, given k, if it satisfies both the IS and ES conditions.

Given h^M, the payoffs of the players, and therefore their incentive to form leagues, is characterized entirely in terms of the tuple (k, j). The tuple (k^*, j^*) is an *equilibrium* of the league formation game if the coop league k^* is stable given j^*, and if the league of firms j^* is stable given k^*.

3. Technical preliminaries

It will be convenient in the following discussion to define the following functions:

$$\lambda(k, j) \equiv \frac{\Pi^M(k, j)/\mu(h^M) - \Pi^{NM}(k - 1, j)/\mu(h^M)}{k - 1}, \qquad \Lambda(k, j) \equiv \frac{\hat{\Pi}^M(k, j) - \hat{\Pi}^{NM}(k, j - 1)}{j - 1},$$

$$\xi(k, j) \equiv \frac{\Pi^M(k + 1, j)/\mu(h^M) - \Pi^{NM}(k, j)/\mu(h^M)}{k}, \qquad \Xi(k, j) \equiv \frac{\hat{\Pi}^M(k, j + 1) - \hat{\Pi}^{NM}(k, j)}{j} \tag{11}$$

The IS and ES conditions and the equilibrium can now be described in terms of these functions. It will be notationally convenient from now on to let $\phi(l) \equiv c(l)/l, l \geq 1$. Then (k^*, j^*) is an equilibrium of the league formation game if:

$$\lambda(k^*, j^*) \geq \phi(k^* - 1), \qquad \xi(k^*, j^*) \leq \phi(k^*), \qquad \Lambda(k^*, j^*) \geq \phi(j^* - 1), \qquad \Xi(k^*, j^*) \leq \phi(j^*) \tag{12}$$

We collect the properties of these functions in the following lemma.[8]

Lemma 1. *Consider the set of coops. Then for any j:*

a. $\lambda(., j)$ and $\xi(., j)$ are continuously differentiable on the set of reals for $k > 1$.
b. $\partial \xi / \partial k \lessgtr 0$ as $k \gtrless k_1$ where $k_1 = (N + 1)/4$.
c. $\partial \lambda / \partial k \lessgtr 0$ as $k \gtrless k_2$ where $k_2 = (N + 5)/4$.
d. $\xi(1, j) > \lambda(1, j), \xi(n, j) < \lambda(n, j)$, and $\xi(k_1, j) = \lambda(k_2, j)$. λ and ξ are strictly decreasing in j. Further, $\xi(k_3, j) = \lambda(k_3, j)$ for $k_3 = (N + 3)/4$.
   Now consider the set of firms. Then for any k:
e. $\Lambda(k,.)$ and $\Xi(k,.)$ are continuously differentiable on the set of reals for $j > 1$.
f. $\partial \Xi / \partial j \lessgtr 0$ as $j \gtrless j_1$ where $j_1 = (N + 1)/4$.
g. $\partial \Lambda / \partial j \lessgtr 0$ as $j \gtrless j_2$ where $j_2 = (N + 5)/4$.
h. $\Xi(k, 1) > \Lambda(k, 1), \Xi(k, n) < \Lambda(k, n)$, and $\Xi(k, j_1) = \Lambda(k, j_2)$. Λ and Ξ are strictly decreasing in k. Further, $\Xi(k, \hat{j}_3) = \Lambda(k, \hat{j}_3)$ for $\hat{j}_3 = (N + 3)/4$.

Let us define:

$$\bar{\delta} = \frac{[\mu(h^M) - 1](\alpha - \gamma_0)}{\mu(h^M)n + m + 1} \tag{13}$$

It is easily verified that:

Lemma 2. *If $\delta < \bar{\delta}$, then for any l:*

$$\Lambda(l, l) > \lambda(l, l), \qquad \Xi(l, l) > \xi(l, l) \tag{14}$$

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[8] In the endogenous coalition formation literature, it is customary to obtain the equilibrium league size through differential techniques that implicitly lets the league size vary continuously on the set of reals. If the equilibrium league size, say l^*, derived through differentiation is not an integer, then the equilibrium league size is simply taken as $[l^*]$, the largest integer that is smaller than l^*.
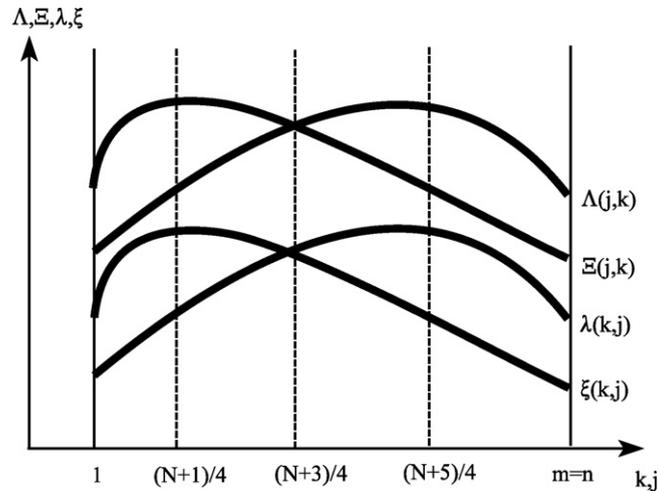
Fig. 1. Functions depicting IS and ES conditions for coops and firms.

The coop has a lower marginal cost of production than a firm by the amount δ. Therefore a firm has a lower Cournot profit than a coop. On the other hand, coops have greater league formation costs. Consider the situation where both set of players are symmetrically placed, i.e. $k = j = l$. For a given h^M, the incentives of firms to form a league (as captured by the IS condition) will dominate that of decision-makers in coops if the cost reduction δ is not too large. Lemma 2 makes it explicit what "not too large" means by putting an upper bound on δ in terms of the parameters of the second-stage game.

The four functions are shown in Fig. 1 under the assumption that $\delta < \bar{\delta}$. It is interesting to note that λ and Λ attain their maximum at the same value of the league size, i.e. at $(N + 5)/4$. The same is true of ξ and Ξ.

Now consider, say, the coops (an analogous argument applies to firms as well). Given j, the minimum condition required for a coop league to be viable is that for some $k' > 1$ we have $\lambda(k', j) \geq \phi(k' - 1)$, i.e. k' meets the IS condition. If this is not true, then $\lambda(k, j) < \phi(k - 1)$ for all k and no league will form. Of course, a k' satisfying just the IS condition may not be stable because it is possible that $\xi(k', j) > \phi(k')$ and outside coops want to enter the league. However, as non-members join the league, this process moves in the direction of *increasing* the league size. It will either end with the grand coalition of all coops (if $\lambda(n, j) \geq \phi(n - 1)$) or some $k'' < n$ (where $\lambda(n, j) < \phi(n - 1)$). In either case we have a non-trivial stable league. Now note that an increase in j shifts the function λ (as well as the function ξ) *downwards* for each k. Because the cost of league formation is not affected, the increase in j makes it harder for coops to satisfy the IS condition. For example, suppose $\lambda(n, j) = \phi(n - 1)$ and thus a grand coalition of coops is just possible with a league of j firms. However, $\lambda(n, j') < \phi(n - 1)$ for $j' > j$ and the grand coalition is no longer stable when the league of firm expands by $j' - j$ members. Therefore, the formation and expansion of a league by one set of players imposes a negative externality on the other set by lowering their incentives to form a league. The intuition is straightforward: an expansion of a league decreases the marginal cost of production for both existing and new league members; in the Cournot model, this results in a decrease in the equilibrium output and profits of all players who do not belong to the league in question.

4. Characterization of the equilibrium league sizes

In this section we characterize the equilibrium of the league formation game for both general and linear costs of league formation.

4.1. General league costs

We start with a general specification of league costs and state our assumptions on these costs directly in terms of restrictions on the average league cost function $\phi(k) \equiv c(k)/k, k \geq 1$. Note that $\phi(k)$ is defined for $k \geq 1$ because $k = 1$ when all players are singletons and $k > 1$ when a non-trivial league is formed.

Assumption A. The real-valued function ϕ is continuously differentiable for $k \geq 1$. Further for any given m, n, h^{M}:

1. There exists $k' > (N + 3)/4$ such that $\phi(k)$ is non-increasing for $k < k'$ and non-decreasing for $k \geq k'$.
2. There exists j_n such that:

$$j_n = \min\{j : \Lambda(n, j) = \phi(j)\} < \hat{j}_3$$

3. There exists k_m such that:

$$k_m = \min\{k : \lambda(k, m) = \phi(k)\} < k_3$$

where k_3 and $\hat{j}_3$ were defined in Lemma 1.

Note that $\phi(k)$ is defined for $k \geq 1$. The case $k = 1$ corresponds to the case where no leagues are formed. These assumptions on league costs rule out the trivial case where league costs are so high relative to Cournot profits that no leagues are possible. In addition, they also ensure the existence of equilibrium (see Proposition 2).

Start with the situation where no leagues have been formed. Let us define the thresholds:

$$k_\tau = \min\{k > 1 : \lambda(k, 1) = \phi(k - 1)\}, \qquad j_\tau = \min\{j > 1 : \Lambda(1, j) = \phi(j - 1)\}$$

Set $k_\tau = 1$ if $\lambda(k, 1) > \phi(k - 1) \, \forall k > 1$ and similarly $j_\tau = 1$ if $\Lambda(1, j) > \phi(j - 1) \, \forall j > 1$. Any league of players, say a coop league, will have to exceed the threshold, k_τ, in order to be viable. We can now prove:

Proposition 1. *Under* Assumption A, *the thresholds k_τ and j_τ exist. Further, $j_\tau \leq k_\tau$. The inequality is strict if $j_\tau > 1$.*

Proposition 1 states that from the situation where all players are singletons, it will be easier for firms to form a league relative to coops because they have a lower threshold to cross. Once firms form a league, then the presence of negative externalities further vitiates the incentives of a coop to agglomerate. It is this combination of lower thresholds and negative externalities that, while allowing a league of firms to be successful, restricts, and possibly rules out, a profitable coop league. We have yet to show that equilibrium leagues exist. This concern is addressed in the following:

Proposition 2. *For any given h^{M}, let $\Psi^*(h^{\mathrm{M}})$ denote the set of equilibrium league structures (k^*, j^*). Under* Assumption A, *an equilibrium league exists, i.e. $\Psi^*(h^{\mathrm{M}}) \neq \emptyset$.*

In the framework of general costs, we can also get equilibria in which no coop leagues are formed at all by relaxing Assumption A.3. In particular, suppose that there exists $j' \geq 1$ such that $\lambda(k, j) < \phi(k - 1) \forall k$ and $\forall j \geq j'$. Then following the same argument as in Proposition 2, we can show the existence of equilibria of the type $(1, j^*)$ where the coops remain as singletons and only a league of firms of size $j^* > j'$ is formed. This can be seen quite transparently by considering the case of linear league formation costs.

4.2. Linear league cost

We will now assume that the cost to a decision-maker of being a member of a league of size $k > 1$ and coordinating with $k - 1$ other decision-makers is given by $c(k - 1) = f(k - 1)$ where $f > 0$ and $k > 1$. Therefore:

$$\phi(k - 1) = \frac{c(k - 1)}{k - 1} = f, \quad k > 1 \tag{15}$$

The assumption that ϕ is a constant allows us to obtain a sharper characterization of the second-stage equilibria. This is because, keeping f fixed, we can use Lemma 1 to examine how the incentives of any one set of players to form a
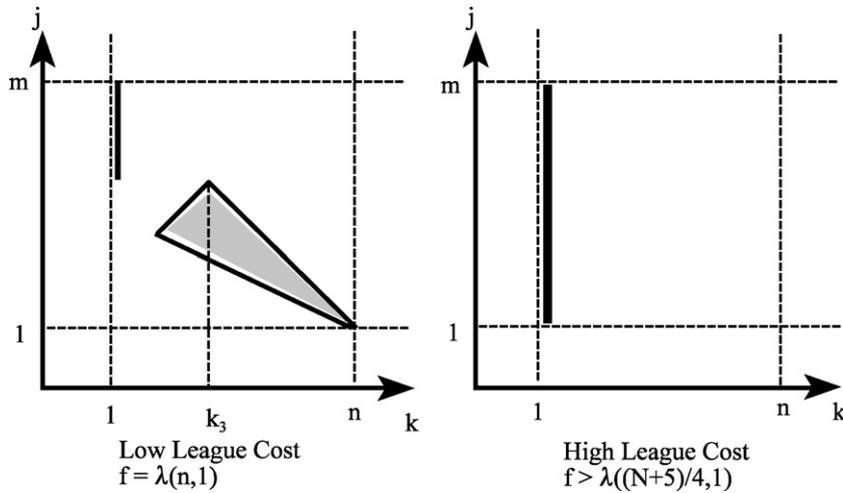
Fig. 2. Stable league sizes for coops.

stable league varies with the size of the league of the other set (i.e. the extent of negative externality exerted by one league type on another).

Let us consider the case of coops first. The set of possible stable coop leagues as a function of j, the size of the league of firms, is shown in Fig. 2.

Let us start with the case shown on the left that corresponds to low values of f. In particular, let us fix the value of f at $f = \lambda(n, 1)$. It will be convenient to define the following league benchmark functions:

$$\underline{k}(j) = \min\{k : \xi(k, j) \leq f, \lambda(k, j) \geq f\}, \qquad \bar{k}(j) = \max\{k : \xi(k, j) \leq f, \quad \lambda(k, j) \geq f\}$$

Recall that an increase in j results in a parallel downward shift in λ and ξ. We can now observe the following:

1. Let $j = 1$, i.e. there is no firm league. Since $\lambda(n, 1) = f > \xi(n, 1)$, the IS condition is just binding and the ES condition does not hold. All coops can profitably cover their league costs. Therefore, all coops have an incentive to join the league and we have $\underline{k}(1) = n$.
2. Now let $\tilde{j}$ be such that $\lambda((N + 3)/4, \tilde{j}) = f$. Over the range $1 < j < \tilde{j}$, the ES condition determines $\underline{k}(j)$ while the IS condition determines $\bar{k}(j)$. Both these values decline as j increases and thus the stable coop league sizes that can be sustained also decrease.
3. Now let j' be such that $\lambda((N + 5)/4, j') = f$. The range $\tilde{j} < j < j'$ is distinguished by the fact that $\underline{k}(j)$ and $\bar{k}(j)$ are both determined by the IS condition. For these values of j, non-member coops have no incentive to join the league and we only need to ensure that member coops have no incentive to exit the league. Given the particular shape of $\lambda(k, j)$, we observe a *non-monotonic* change in the size of the stable league as j increases: $\underline{k}(j)$ increases and $\bar{k}(j)$ falls with an increase in j. Therefore, smaller and larger coop leagues can no longer be sustained and the stable number of coops in the league converges towards the intermediate size $k_3 = (N + 3)/4$; in fact, when $j = j'$, the stable coop league size is unique and equal to k_3.
4. When $j > j'$, the league of firms is sufficiently large to ensure that no stable coop league will form.

The negative externalities property is reflected in the fact that larger leagues of firms are generally associated with smaller stable coop leagues. An interesting exception is the non-monotonicity property observed for an intermediate-sized league of firms. For these intermediate values of j, no non-member coop wishes to join the league. Therefore, the stability of the coop league is dictated by the condition that member coops should not exit the league. Given the negative externalities imposed by intermediate-sized league of firms, the only way to ensure that coop members can profitably cover their league costs is by forming larger leagues.

The figure shown on the right in Fig. 2 corresponds to relatively high values of f, for example $f > \lambda((N + 5)/4, 1)$. In this case, $\lambda(k, j) < f \forall k, j$. Therefore, the only stable outcome is where all coops are organized as singletons, i.e. $k = 1$ for all possible values of j.
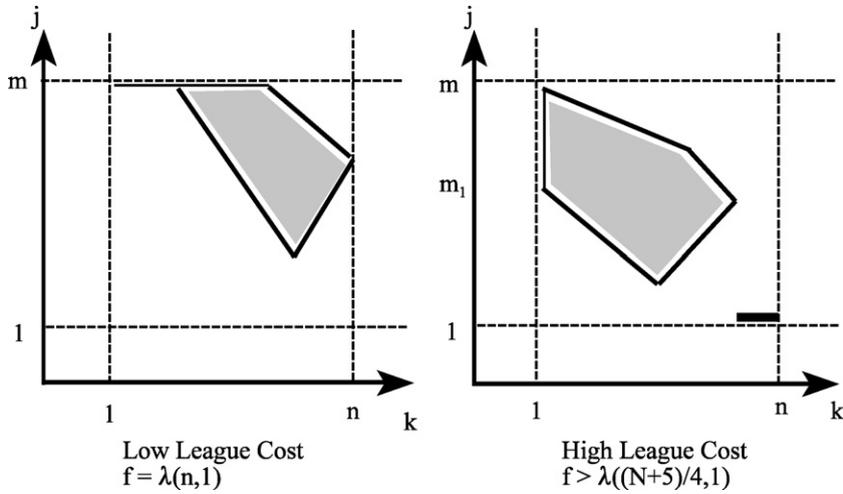
Fig. 3. Stable league sizes for firms.

Fig. 3 shows the set of league of firms that is stable for different values of k, the size of the coop league. The figure on the left corresponds to low costs, i.e. $f = \lambda(n, 1)$. Since it is now possible that $\Lambda(m, k) > f$ for $k > 1$, we see that the grand coalition of firms can be stable even when coops are organized into a league. As k becomes larger, the same considerations as those for coops, including the non-monotonic property, also applies to the league of firms. Note that it is possible that $\Lambda(m, n) > f$ in which case $j = m$ for all k. The figure on the right considers the case of high cost, $f > \lambda((N + 5)/4, 1)$. Now the grand coalition of all firms may only be possible when no coop league is formed. In addition, if $\Xi(m, 1) < f$, then the ES condition is binding and leagues less than size m can also be stable. For high values of k, it is possible that only the singleton firm league is stable.

Putting Figs. 2 and 3 together give us the set of equilibria Ψ^* of this second-stage game. These are shown in Fig. 4. For low f, Ψ^* is shown by the shaded polygon and the point $(1, m)$. For high f, it is given by the heavily shaded line
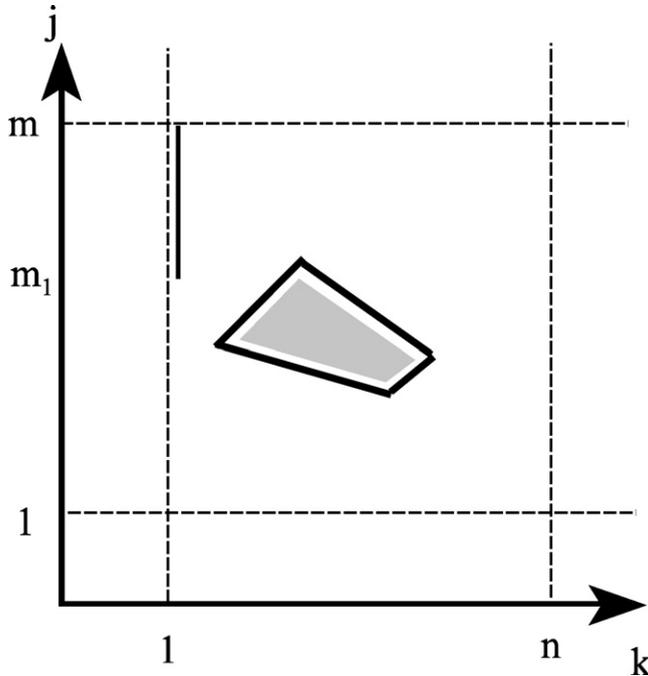

Fig. 4. Equilibrium league sizes.

$(1, m')$, $m_1 \leq m' \leq m$. Therefore, in many cases it is possible to have an equilibrium in which all firms form a league and all coops remain singletons. For the cases where a non-singleton coop league forms, as shown by the polygonal area, a non-singleton league of firms will be stable as well and generally larger in size than the coop league. Note that what drives this result is the fact that firms have a lower threshold to cross in order to form a stable league and, once formed, exert a negative externality on the incentives of coops to form a league. These negative externalities on a coop's incentives are further reinforced by higher costs of league formation. We now consider a concrete example:

Example 1. Let us suppose that $n = 10$, $m = 9$, $\delta = 1$, $\alpha = 419$, $\gamma_0 = 19$, $\gamma = 1$ and $\mu(h^M) = h^M = 3$. It can be verified that these values satisfy all the assumptions of the model. Letting $f = 12.57$, the following table lists the values of k that are stable for a given j:

j	k
1	10
2	9
3	9
4	7, 8
5	1
6	1
7	1
8	1
9	1

From the table we can see the negative externality property: as the league of firms increases in size, the size of the coop league falls. When the league of firms is sufficiently large ($j = 5$), the coop league is not formed. It can similarly be verified that $\Lambda(k, j) > f$ for all k. Thus the only stable league of firms is the grand coalition of all firms. It then follows that the equilibrium of the league formation game is $\Psi^*(h^M) = (1, 9)$ where all coops remain singletons and all firms form a league.

It is important to note that the non-existence of a coop league is *not* solely due to a coordination problem among the decision-makers that prevents the first k_τ coops from announcing a league. Consider once again the case where f is low and the polygonal area shows that a non-trivial equilibrium with non-singleton coop league and league of firms exists. If the problem was one of coordination, then one could in principle consider refinements of Nash equilibria based on profitable coalitional deviations that would give us non-trivial leagues. Of course one problem in the real world is that such a coordinating mechanism may not exist. This has been provided as one reason why coop leagues are not widely prevalent. Our analysis points out that even if such a mechanism did exist and was available to coops, then in principle it would be available to firms as well. Once firms can coordinate their coalitional deviations, they will have an incentive to form a league first and, by virtue of negative externalities, force the coops to remain singletons. Thus refinements based on coordinated coalitional deviations will not help to resolve the problem of non-existence of the coop league.

5. The coop formation game

We can now backwardly induct to the first stage and consider the coop formation game. There are three things to be determined here: the number of coops that are formed, how many of these coops will join the league, and the size of member and non-member coops. Since the exclusive membership game is static, all coops that will be members of the coop league (if one forms) will be of the same size while all coops that will be out of the league will be symmetric too and of identical size. We can therefore look at a representative member and non-member coop when discussing the incentives of their constituent agents. Before analyzing these incentives, we have to resolve a technical issue. If the set of equilibria of the league formation game, Ψ^*, has more than two elements, then it creates a coordination problem for agents. Depending on the particular league structure which is assumed to be the focal point, we can have multiple subgame perfect equilibria. Our solution to this problem is to bias the focal point towards the largest coop league, i.e. we assume, for any given h^M, that agents coordinate on the *largest* possible equilibrium coop league:

$$\bar{k}^*(h^M) = \max\{k^* : (k^*, j^*) \in \Psi^*(h^M)\}$$

Correspondingly, since the incentives of firms are similar, we let firms coordinate on

$$\bar{j}^*(h^M) = \max \left\{ j^* : (\bar{k}^*(h^M), j^*) \in \Psi^*(h^M) \right\}$$

By first choosing the largest *coop* league among all possible equilibria as the focal point, we are in fact able to *strengthen* our result on the non-formation of coops and coop leagues.

The coop cost function is assumed to satisfy:

Assumption B. The real-valued function v is continuously differentiable for $h \geq 1$. Further, there exists $\tilde{h}$ such that $v(h)/h$ is decreasing for all $h \leq \tilde{h}$ and increasing for all $h > \tilde{h}$.

Fix n, the number of coops. We first examine how h^M and h^{NM} are determined as functions of n. We will then see how n is determined by a "zero profit" condition. Each of the agents in a coop of size h is assumed to receive the fraction $1/h$ of the coop's profits. That is, for simplicity we assume the egalitarian version of the coop as in the labor-managed firm literature (Meade, 1972). For an agent who is in a coop of size h that belongs to a league, the reduced form profits are

$$\pi^M(h) = \frac{1}{h} \left[\Pi^M(\bar{k}^*(h), \bar{j}^*(h)) - \mu(h)c(\bar{k}^*(h) - 1) - v(h) \right] \tag{16}$$

while that of a non-member in a coop of size h' is equal to

$$\pi^{NM}(h') = \frac{1}{h'} \left[\Pi^{NM}(\bar{k}^*(h), \bar{j}^*(h)) - v(h') \right] \tag{17}$$

We now focus attention on the incentives of agents to form coops when they can look forward into the game and anticipate the equilibrium of the second-stage league formation game and its consequences for the product market competition with firms and other coops. From now on we will restrict the discussion to the case of linear league formation costs.

Let us start with agents in the representative member coop. Suppose there is an increase in h, the size of the coop. This leads to an increase in $\mu(h)$ and thus shifts down the functions $\lambda(k, j)$ and $\xi(k, j)$ for any given j. This leads to a smaller coop league $\bar{k}^*(h)$ and, by virtue of negative externalities, a larger firm league $\bar{j}^*(h)$. Thus the member coop's profits and each agent's profit share falls. However, since the per agent coop formation cost $v(h)/h$ is decreasing in h, these "scale economies" from coop size could outweigh the decline in product market profits and an agents profit, $\pi^M(h)$, could be initially increasing in h. As h continues to increase however, the per agent cost will begin to rise and $\pi^M(h)$ will now start to fall with h. This leads to an "inverted U-shaped" reduced form profit curve as shown in Fig. 5.

Similar considerations apply to agents in the representative non-member coop. The only difference is that an increase in h for a non-member coop does not influence the league structure (k, j) since the IS and ES conditions for coops (and firms) is not affected by the size of non-member coops. Therefore profits could be increasing with h over a larger range of values than for member coops. Recall from the IS condition that for any league structure (k, j):

$$\Pi^M(k, j) - \mu(h)c(k - 1) \geq \Pi^{NM}(k - 1, j) > \Pi^{NM}(k, j)$$

where the last inequality follows from the fact that Π^{NM} is strictly falling with k. Thus, if a coop league exists, then for the same size h, $\pi^M(h) > \pi^{NM}(h)$. However, if h is sufficiently large so that no coop league is formed, then $\bar{k}^*(h) = 1$ and for these values of h we have $\pi^M(h) = \pi^{NM}(h)$.

Agents will choose the size of their coop to maximize their per-member profits.[9] Thus agents in member coops will choose the size h^M (or more precisely $[h^M]$) such that $d\pi^M/dh = 0$. Likewise agents in non-member coops will choose their size h^{NM} such that $d\pi^{NM}/dh = 0$. The formation of these coops proceeds through the rules of the exclusive membership game. Agents can then compare their profit share as part of a coop with the exogenous wage w. If their profit share is greater than w, then each agent has no incentive to deviate from his message and break up the coop into singleton agents. On the other hand, if w is greater, then each agent has an incentive to withdraw from the coop by changing his message and accepting w.

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[9] Note that coop members choose the coop size to maximize per-member profits. Dow (2003, Chapter 7) has noted that if side payments are allowed between insiders and outsiders in a coop, then a coop can also be visualized as maximizing total profits.
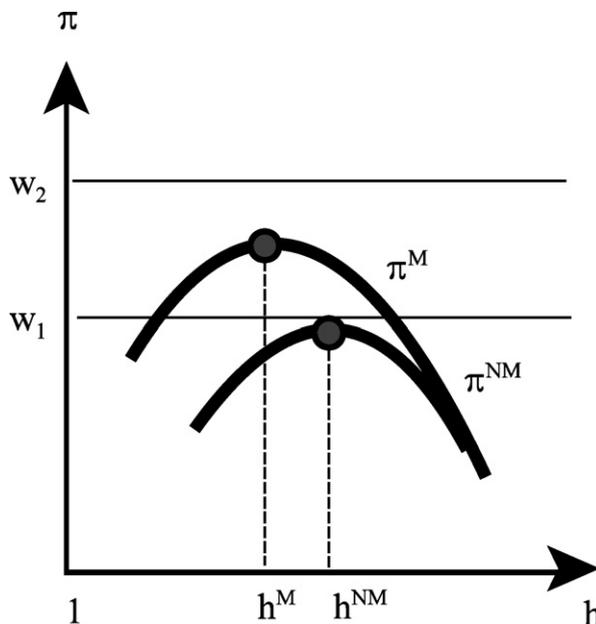
Fig. 5. Coop formation game.

Consider the marginal profits of non-member coops at the coop size h^M. For agents in the member coops, the marginal benefit from expanding the coop size is equal to the marginal cost at h^M. However, for agents in the non-member coop, the marginal cost is lower because its size has no direct impact on the second-stage league formation game and therefore no adverse impact on its profits. Therefore, $(d\pi^{NM}/dh)|_{h^M} > 0$ and a non-member coop will have a larger number of agents than a member coop. Agents in coops that anticipate belonging to a league have an incentive to keep their size relatively smaller by being more exclusive in order to reduce their coordination costs in the second stage and facilitate the formation of a coop league.

It finally remains to determine n, the total number of coops. Note that an increase in n lowers both π^M and π^{NM}, and thus possibly h^M and h^{NM}. Let us now parameterize profits and membership size by n to denote this dependence. Suppose that for some n, $w < \pi^{NM}(h^{NM}(n), n) \le \pi^M(h^M(n), n)$. Then agents will form coops and enter the industry until for some n^*, $w = \pi^{NM}(h^{NM}(n^*), n^*)$. Now there is no incentive for outside agents to form coops and enter. Given $h^M(n^*)$, the equilibrium league structure in the second stage will be $\bar{k}^*(h^M(n^*))$ and $\bar{j}^*(h^M(n^*))$. Thus $\bar{k}^*(h^M(n^*))$ coops will be league members and $n^* - \bar{k}^*(h^M(n^*))$ coops will remain outside the league. The agents in non-member coops receive a payoff equal to their outside option while those in member coops earn strictly more. It is a subgame perfect equilibrium because, looking forward to the second stage, no non-member coop wishes to join the league, and no agent within each coop has an incentive to change his message. This is shown in Fig. 5 for the wage w_1. If $\pi^M(h^M) < w$ for all n (as shown by wage w_2 in Fig. 5), then no coops will form at all. The equilibrium coop league in the second stage cannot engender sufficient profits in order to make member coops competitive with respect to the league of firms and cover the outside option of the agents. We can now state the main result of this section:

Proposition 3. *Suppose w is given and* Assumption B *holds. In the subgame perfect equilibrium*:

a. *If $w \le \pi^{NM}(h^{NM}(n), n)$ for some n, then both member and non-member coops exist in the industry. The equilibrium number of coops in the industry, n^*, is determined by the condition $w = \pi^{NM}(h^{NM}(n^*), n^*)$. There are $\bar{k}^*(h^M(n^*))$ member coops in a league and $n^* - \bar{k}^*(h^M(n^*))$ non-member coops. These n^* coops compete in the product market with a firm league of $\bar{j}^*(h^M(n^*))$ member firms and $m - \bar{j}^*(h^M(n^*))$ non-member firms.*
b. *If $\pi^{NM}(h^{NM}(n), n) < w$ for all n and $w \le \pi^M(h^M(n), n)$ for some n, then only those coops can exist that are members of a league. The equilibrium number of coops in the industry, n^*, is determined by the condition $w = \pi^M(h^M(n^*), n^*)$. There are n^* member coops in a league, each with $h^M(n^*)$ members, competing in the product market with a firm*

league of $\bar{j}^*(h^M(n^*))$ member firms and $m - \bar{j}^*(h^M(n^*))$ non-member firms. No non-member coops exist in the industry.

c. *If $\pi^M(h^M(n), n) < w$ for all n, then no coops will form and the industry is composed only of firms.*

The last point highlights one of the main results of the paper. Agents in member coops correctly anticipate that the league they will form subsequently will not be large enough. It is possible that for *non-equilibrium* sizes of the coop league, agents in member coops could have earned more than their outside option. For example, it is possible that for some sufficiently large k' (and correspondingly some value of n), agents in member coops could earn more than the wage. Thus member coops, each with a marginal cost of production less than that of corresponding firms, could exist in the industry if they could form a coop league of size k' or larger. However, firms have an incentive to form a league first and the ensuing negative externalities implies that a coop league of sufficient size such as k' may not emerge *in equilibrium*. It then follows that no coops are formed in equilibrium because agents correctly foresee that they will not be able to subsequently agglomerate into a sufficiently large coop league.

6. Efficiency

In this section our aim is to show that a coop league may not form even if efficiency dictates that such a league exist. Accordingly, suppose there are n symmetric coops with h members each. The league structure (k, j) is *efficient* if it maximizes aggregate welfare defined as the sum of consumer surplus and the aggregate profits of both sets of players (net of any costs of coop and league formation). Let $q(k, j)$ and $\hat{q}(k, j)$ denote the aggregate output produced by coops and firms, respectively. Then total output is equal to

$$Q(k, j) = q(k, j) + \hat{q}(k, j) = \frac{1}{N+1} \left[N(\alpha - \gamma_0) + n\delta + \gamma k(k-1) + \gamma j(j-1) \right] \tag{18}$$

In our model aggregate welfare, $W(k, j)$, is given by

$$W(k, j) = \tfrac{1}{2}[Q(k, j)]^2 + k \left[\Pi^M(k, j) - \mu(h^M)\phi(k-1) \right] + (n-k)\Pi^{NM}(k, j)$$
$$+ j \left[\hat{\Pi}^M(k, j) - \phi(j-1) \right] + (m-j)\hat{\Pi}^{NM}(k, j) - kv(h^M) - (n-k)v(h^{NM}) \tag{19}$$

It is difficult to provide general results on efficiency given the complexity of the terms involved. The most general result that can be provided is the following:

Proposition 4. *Let $n = m$. The league structure (n, n) is efficient for sufficiently small costs of coop and league formation and sufficiently small marginal cost reduction δ.*

The proof follows Yi (1998, Proposition 5) and is therefore omitted. An examination of (18) shows that aggregate output is strictly increasing with an increase in the size of both leagues. Therefore consumer surplus is maximized at (n, n). The situation with aggregate profits is more ambiguous. There are four distinct groups when $k > 1$ and $j > 1$: member coops, non-member coops, member firms and non-member firms. An increase in the size of any one league increases the profits of those league members but adversely affects the profits of the remaining three groups. The problem of comparing the net change in aggregate profits is further compounded by the fact that the marginal cost of production differs for firms and coops. Finally, even if it can be shown that the increment in consumer surplus plus the change in aggregate profits is positive, there still remains the issue of accounting for the costs of coop and league formation. Proposition 4 is therefore able to provide a general result only for the case where the difference in marginal costs and the costs of coop and league formation are not too large. If these costs are large, then league structures other than (n, n) could be efficient. In particular, it is possible that the only efficient league structure is one where coops are organized into a league and the firms are singletons. Moreover, it is also possible that the equilibrium outcome differs from the efficient one. For instance, while it is efficient that only a coop league is formed, an equilibrium could be one where only firms are organized into a league. In order to highlight these possibilities, we now consider a concrete example.

Example 2. Let us suppose that $n = m = 2$. There are now four possible league structures: $(1, 1), (1, 2), (2, 1)$ and $(2, 2)$. We will let $\delta = 1, \alpha = 20, \gamma_0 = 4, \gamma = 1$ and $\mu(h^M) = h^M = h^{NM} = 3$. It can be verified that these values satisfy all the parametric restrictions assumed in the model. The aggregate welfare corresponding to the four league structures are

$$W(1, 1) = 57.76 - 2\nu(3), \qquad W(1, 2) = 59.84 - 2\phi(1) - 2\nu(3) \qquad W(2, 1) = 63.84 - 6\phi(1) - 2\nu(3),$$

$$W(2, 2) = 64 - 8\phi(1) - 2\nu(3)$$

We can now determine the efficient league structure as a function of ϕ:

Costs, ϕ	Efficient (k, j)
$0 \leq \phi(1) \leq 0.08$	$(2, 2)$
$0.08 < \phi(1) < 0.85$	$(2, 1)$
$\phi(1) > 0.85$	$(1, 1)$

Therefore it is efficient at low league costs for all players to form their respective leagues while, at high league costs, all players should remain singletons. At intermediate values of league costs, only one league (in this case the coop league) should form.

It is interesting to note that at intermediate league costs, $0.08 < \phi(1) < 0.85$, the league structure $(1, 2)$ is an equilibrium. In the case of firms, $\hat{\Pi}^M(1, 2) = 11.56 - \phi(1) > \hat{\Pi}^{NM}(1, 1) = 7.84$. Thus no league member has an incentive to exit. Because of the implicit coordination problem, coops cannot form a league: an individual coop cannot *unilaterally* form a league by announcing *Yes* if the other coop has announced *No*. Thus we have a conflict between the equilibrium and efficient outcomes. While efficiency dictates that only a coop league should form, the equilibrium is one where only a league of firms is established.

It is also important to note that the presence of a coordinating mechanism that allows profitable coalitional deviations will not help to resolve this tension between equilibrium and efficiency. Suppose $0.08 < \phi(1) < 0.85$ and we start from the structure $(1, 1)$. Let us suppose that there is a coordinating mechanism whereby the coops can form a league. Now consider the structure $(2, 1)$. It is efficient over this range of costs. It can be checked that $\Pi^M(2, 1) = 19.36 - 3\phi(1) > \Pi^{NM}(1, 1) = 14.44$, so that the coops have no incentive to exit from the league. However $(2, 1)$ is no longer robust against coalitional deviations. In an ex ante symmetric framework, if a coordinating mechanism is available to coops, it should be available to firms as well. Using such a mechanism, firms will now find it profitable to form a league because $\hat{\Pi}^M(2, 2) = 9 - \phi(1) > \hat{\Pi}^{NM}(2, 1) = 5.76$. Therefore, the equilibrium is $(2, 2)$ which is not efficient. $\qquad \square$

7. Conclusion

In this paper, we have generalized from features found in the two most prominent coop leagues – Mondragon and La Lega – to develop a formal game-theoretic model to explain the relative rarity of coops and coop leagues. In constructing this model we have used current developments in the theory of endogenous coalition formation. Our paper highlights the differences between firms and coops that condition their respective incentives to form a league. It shows why firms may form a league even when their marginal costs are higher than coops and even when it is efficient from a welfare point of view that only a coop league should operate. We then demonstrated why agents, anticipating the non-existence of a coop league, are deterred from organizing themselves into coops.

There are a number of avenues for future research that remain to be explored. For example, we can consider a richer model of coop formation in the first stage. One possible organizational advantage that decision-makers could have from forming a coop is that peer monitoring may mitigate the moral hazard problems stemming from the non-observability of effort (Ireland and Law, 1988). The incentives to form coops in the presence of moral hazard issues could then be analyzed along the lines of Espinosa and Macho-Stadler (2003).

The coalition formation games in this paper are essentially static in that they are based on simultaneous-move coalition announcement games. It would also be interesting to extend the analysis to an explicitly dynamic framework where players move sequentially to form coalitions. It is an important issue whether in a dynamic game with far-sighted players we can have an efficient coop league. On the one hand it seems that players would have an incentive to form such a league since they can look forward and calculate the incentives of other players to participate in this league once it has exceeded some critical threshold. On the other hand, if the cost of forming such a league is borne primarily by

the early movers, then players would have an incentive to wait and let others go first. All these issues can be properly addressed in a dynamic setting (for instance, Adsera and Ray, 1998; Dutta et al., 2005).

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Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at doi:10.1016/j.jebo.2008.04.001.

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